EFB 462/662 Animal Physiology: Environmental & Ecological
How much water do I need?

One can use Henry's Law and some simple stoichiometry to estimate how much water a gill must process to extract oxygen.

We remember first that there is a strict stoichiometry that governs the oxidation of glucose to carbon dioxide, water and energy:

$$C_6H_{12}O_6 + 6\ O_2 \rightarrow 6\ CO_2 + 6\ H_2O + 2.82\ \text{MJ (mol glucose)}^{-1}$$

We may use the stoichiometry to formulate a molar energy production rate. The molar energy production for glucose, for example, is 2.82 MJ (mol glucose)$^{-1}$. Let us designate this is Q_{Gl}. There is also a molar energy production for oxygen: because 6 moles of oxygen are consumed for every mole of glucose consumed, oxygen's molar energy production (Q_{O2}) is one-sixth of that for glucose, or 0.47 MJ (mol O_2)$^{-1}$:

We remember also the fundamental definition of metabolic rate as a *power*, that is, a *rate* of energy utilization per time. We express this with the symbol $\dot{Q}$. In SI units, the unit of power is the *watt* defined as an energy usage rate of 1 1 J s^{-1}. An energy production rate, therefore, implies a molar consumption rate for both glucose and oxygen. Let us designate a molar consumption rate as $\dot{M}$, which has units of mol s^{-1}.

The glucose (or oxygen) necessary to yield 1 joule is simply the inverse of the molar energy production. For glucose, it is:

$$\frac{M_{Gl}}{2.82MJ} \cdot \frac{1MJ}{10^6 J} = 3.54 \cdot 10^{-7}\ \frac{M_{Gl}}{J}$$

For oxygen, it is:

$$\frac{M_{O2}}{0.47MJ} \cdot \frac{1MJ}{10^6 J} = 2.12 \cdot 10^{-6}\ \frac{M_{O2}}{J} = 2.11 \frac{\mu M_{O2}}{J}$$

It follows that a particular molar consumption rate, $\dot{M}$ is required to support a particular metabolic rate, $\dot{Q}$. By the above equation, 2.12 μmoles of oxygen must be consumed each second to support a metabolic rate of 1 watt.

How much water is needed to supply oxygen at this rate?

The solubility coefficient for oxygen in water, α_{O2}, at 20°C is about 13.7 μmol l_{H2O}^{-1} kPa^{-1} (SI units). By Henry's law, the molar concentration of oxygen in water is:

$$[O_2] = \alpha O2 * pO2$$

where pO_2 is the oxygen partial pressure. At sea level atmospheric pressure (101.525 kPa), the oxygen partial pressure is the mole fraction of that, or 0.2095 * 101.525 = 21.3 kPa. According to Henry's law, the molar concentration of oxygen will be (13.7 μmol l_{H2O}^{-1} kPa^{-1}) * 21.3 kPa = 291 μmol l^{-1}.

To support a metabolic rate of 1 watt, the volume of water that must be processed each second, $\dot{V}_{H2O}$, (l s^{-1}) is therefore:

$$\dot{V}_{H2O} = \frac{\dot{M}_{O2}}{[O_2]} = \frac{2.12\mu mol \cdot s^{-1}}{291\mu mol \cdot l^{-1}} = 0.0073 \cdot \frac{l}{s} = 7.3 \cdot \frac{ml}{s}.$$

This is a minimum figure, of course, assuming that extraction of oxygen is 100% efficient. If only half the oxygen can be extracted, twice the volume of water must be processed.