

Applied Limnology Problems 2 2006
50 points total

1. Fayetteville Green Lake is a meromictic lake near Syracuse, N.Y. that has been studied extensively both because it provides a source of undisturbed sediments for paleo-limnological work, and because it has an interesting layer of purple sulfur bacteria that live on the lake's chemocline. The lake is 52.5 m deep.

(a) How much energy would it take to mix the chemocline of Green Lake with the surface waters of the lake in the spring, when the mixolimnion of the lake is isothermal at 4°C and mixing? Calculate this using the approximate values for Green Lake shown below.

Depth – z (meters)	Area at that depth – A – m ²	Temperature - °C	Density r at that depth – g/cm ³
0 (surface)	258,300	4	1.900
5	223,200	4	1.900
10	201,300	4	1.900
15	178,900	4	1.900
20	155,000	7	2.540
25	135,600	7	2.540
30	121,900	7	2.540
40	95,600	7	2.650
50	37,700	7	2.700

(7 points total)

To solve this problem you need to estimate the whole lake stability – the amount of energy needed to mix the lake to uniform density:

$$S = 1/A_0 \int_{z_0}^{z_{\max}} (\rho_z - \rho_{\text{average}}) (z - z_{\text{average}}) (A_z) dz$$

where A_0 = the surface area in cm

A_z = the area at some depth z (in cm)

ρ_{average} = the final or mean density that would result if the lake were completely mixed

ρ_z = the density at depth z

z_{average} = the depth (cm) where the final (mixed) mean density exists prior to mixing

z_{\max} = maximum depth in cm

z_0 = surface or zero depth

(1 point for knowing to use this formula)

To calculate the mean density if the lake were completely mixed, you need to calculate the average density of the lake. To do this you can use the truncated cone or another approximation (such as just multiplying the area at the first depth by the depth difference to the next depth – like step approximations) to calculate the volume at each depth, and then calculate the weighted average of the densities to get the density if you mixed the water. Whichever approximation you use does not make much difference to the final

estimated density, which is approximately **2.19 g/cm³**. Water of this density is currently found between 15 and 20 meters deep. If you use a linear interpolation, you would find this depth of average density at **17.3 m**.

FYI, the truncated cone formula for calculating volume is:

$$V = \frac{h}{3} (A_a + A_b + \sqrt{A_a A_b})$$

The calculations to find these values are shown here:

Density	Depth	Volume	Density*m3	Volume	Density/m3
g/cm3	m	trunc cone	trunc cone	rectangles	rectangles
1.9	0	1202682	2285097	1291500	2453850
1.9	5	1060779	2015480	1116000	2120400
1.9	10	949950	1804904	1006500	1912350
1.9	15	834036	1584669	894500	1699550
2.54	20	725960	1843938	775000	1968500
2.54	25	643446	1634353	678000	1722120
2.54	30	1084840	2755494	1219000	3096260
2.65	40	644448	1707787	956000	2533400
2.7	50	12567	33930	37700	101790
	51	7158708	2.19	7974200	2.21
		tot vol.	mean density g/cm3	tot vol.	mean density g/cm3

4 points for these calculations above

So, with this information, the equation shown above:

$$S = \int_{Z_0}^{Z_{\max}} \frac{1}{A_0} (\rho_z - \rho_{\text{average}}) (z - z_{\text{average}}) (A_z) dz$$

Can be used to find the stability. The calculations are shown below

Depth	Depth	Area	Area	Temp	Density	pz-pave	z-zave	(pz-pave)(z-zave)(Az)dz
m	Cm	m2	cm2	C	g/cm3	g/cm3	cm	gcm ⁻¹
0	0	258,300	2,583,000,000	4	1.9	-0.29	-1730	6.47946E+14
5	500	223,200	2,232,000,000	4	1.9	-0.29	-1230	7.96154E+14
10	1000	201,300	2,013,000,000	4	1.9	-0.29	-730	6.39228E+14
15	1500	178,900	1,789,000,000	4	1.9	-0.29	-230	2.38653E+14
20	2000	155,000	1,550,000,000	7	2.54	0.35	270	3.66188E+14
25	2500	135,600	1,356,000,000	7	2.54	0.35	770	1.09633E+15
30	3000	121,900	1,219,000,000	7	2.54	0.35	1270	2.16738E+15
40	4000	95,600	956,000,000	7	2.65	0.46	2270	4.99128E+15
50	5000	37,700	377,000,000	7	2.7	0.51	3270	3.20649E+15
51	5100							1.41496E+16
								5477986.446
								sum
								1/A0*sum
								g-cm cm-2

So, the stability, the energy needed to mix the lake, is $\sim 5,478,000 \text{ g-cm cm}^{-2}$
2 additional points here

(b) Is this a lot of energy or not? In other words, how likely is it that the monomolimnion will mix with the rest of the lake? Explain/justify your answer.

2 points

This is a huge amount of energy. Any comparison of this with other lake stabilities or some other estimate of energy is fine. There were sample values in your reading (Kalff) showing non-meromictic lakes with stabilities of 100s of g-cm/ cm^2 ; this is 1000X more. Any comparison that is correct gets credit. It is very unlikely this lake will mix completely.

(c) If you have been to Green Lake State Park, you may remember that the lake is surrounded by steep hills. How might this affect the likelihood of the lake to mix?

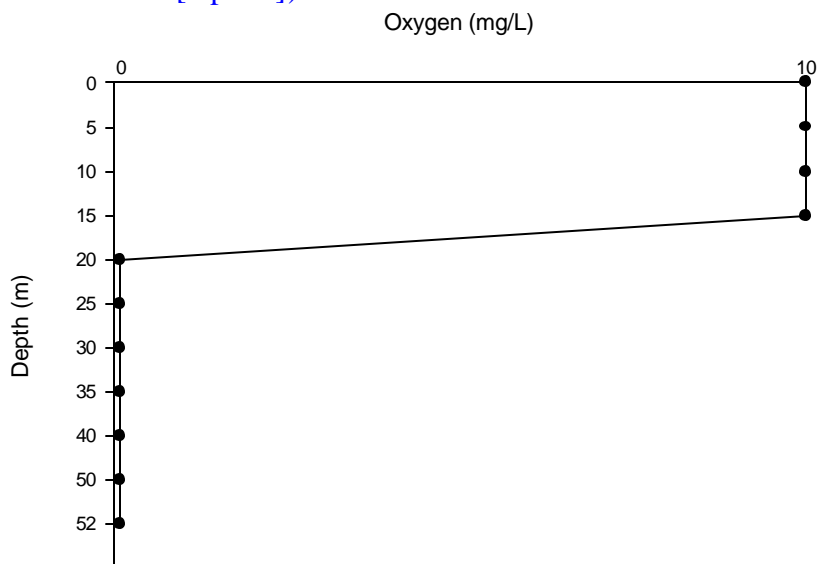
2 points

This *decreases* the likelihood that the lake will mix (1 point) BECAUSE it shelters the lake and reduces the wind speed reaching the surface of the lake. That wind is providing the mixing force, so an even stronger wind would be needed for there to be sufficient wind at the lake surface to mix the lake.

(d) Draw a rough oxygen profile for Green Lake at this time of year (spring), during isothermal mixing in the mixolimnion. Explain why your profile looks the way it does.

5 points total

3 points for the correct profile (need to have axes laid out correctly [1 point] and have no oxygen in the bottom waters [1 point] and uniformly oxygenated waters in the mixolimnion [1 point])



2 points for the explanation – needs to include that the surface waters are oxygenated because they are well mixed and in equilibrium with the atmosphere.

The monomolimnion NEVER mixes and so oxygen has not been replenished for many, many years and this layer is anoxic.

2. You graduate from ESF and become the limnologist for your local town (Stinkville). The sewage from your hometown is treated with secondary treatment and dumped into a local lake that flushes fairly regularly (6X per year). Predictably, with all of the added nutrients and organic matter, your lake's hypolimnion becomes anoxic early each summer, building up large amounts of putrid sulfides. Occasionally there is a storm sufficient to cause upwelling, which allows the stink to reach the surface, causing people to flee their homes temporarily, and slowing tourism.

People have agreed to tertiary treatment to help remediate the lake, but in the short-run the local meteorologist wants you to estimate how strong a storm will be necessary to force upwellings (resulting in the stinky lake burps), so people can be forewarned on the nightly news. Because you are such a good limnologist, you know that many lakes have internal seiches that can cause the thermocline to oscillate, and can result in upwellings.

Your lake is 3.5 km long in the direction of the wind and 25 m deep. In most summers the lake stratifies with 24 degree C epilimnetic water, and an epilimnion thickness of 10 m. The bottom water remains at 4 degrees C throughout the summer. The mean summer wind speeds are about 3.8 m/sec, although normal conditions don't generally result in stinky conditions. You look up the acceleration of gravity (9.8 m s^{-2}) and find a water density table (units of g cm^{-3} ; see last page of problem set) to assist your calculations.

- (a) Based on the parameters above, what is the height of the seiche during average winds in summer?

(6 points – 1 point off for simple math and/or conversion errors)

To calculate the internal seiche height, you first need to know the setup height.

For average winds the given velocity is 3.8 m/sec; the length is 3.5 km = 3,500 m; the z_{max} is given as 25 meters and g is given as 9.8 m/sec^2 :

$$S_h = \frac{(3.2 * 10^{-6})(u^2)l}{gz_{\text{max}}} = \frac{(3.2 * 10^{-6})(3.8\text{m/s})^2(3,500\text{m})}{(9.8 \text{ m/sec}^2 * 25\text{m})} = 0.00066\text{m} = 0.07 \text{ cm}$$

The internal seiche height is then:

$$A_i = S_h \left(\frac{\rho_h}{\rho_h - \rho_e} \right)$$

You can get the densities of the hypolimnion and epilimnion from the table that was attached to the problem set: (1) epilimnion density = density of water at 24 degrees C = 0.9973256 g/cm^3 ; (2) hypolimnion density = density of water at 4 degrees C = 1.0 g/cm^3 .

$$A_i = S_h \left(\frac{r_h}{r_h - r_e} \right) = 0.07 \text{ cm} * (1 \text{ g/cm}^3 / (1 \text{ g/cm}^3 - 0.9973256 \text{ g/cm}^3)) = 24.7 \text{ cm}$$

(b) How big a sustained wind is necessary to get an upwelling in summer?

6 points – again 1 point off for math and conversion errors.

To find this out, you want to know when the internal seiche would be greater than 10 meters (thus breaking the surface). You can get this by rearranging the above equations:

To get the needed setup height for a 10 meter seiche:

$$10 \text{ m} = A_i = S_h \left(\frac{r_h}{r_h - r_e} \right) = X m \left(\frac{1.000000}{(1.000000 - 0.9973256)} \right)$$

$$\text{Rearranging: } X = 10 \frac{(1.000000 - 0.9973256)}{1.000000} = 0.027 \text{ m}$$

Then to get the windspeed, use the setup height equation:

$$Sh = \frac{(3.2 * 10^{-6})(u^2)l}{gz_{mzx}} = \frac{3.2 * 10^{-6} (Y \text{ m/sec})^2 * 3,500 \text{ m}}{9.8 \text{ m/sec}^2 * 25 \text{ m}} = 0.027 \text{ m}$$

rearranging,

$$Y \text{ m/sec} = \sqrt{\frac{(0.027 \text{ m})(9.8 \text{ m/sec}^2)(25 \text{ m})}{3.2 * 10^{-6} * 3,500 \text{ m}}} = 24.2 \text{ m/sec}$$

(c) Describe one other physical, chemical or biological occurrence that likely happens during upwellings, in addition to the stinky burp.

3 points – anything that makes sense is ok here – you are bringing cold, oxygen poor water to one side of the lake – fish and plankton may not be found in that area – will be blown to the other end of the lake; you will have reduced forms of various nutrients and other ions at the surface in the upwelling area; you will start to have oxygen mix into the exposed hypolimnion....

3. “Liming” is one method by which people attempt to remediate the effects of acid precipitation. When a lake management group decides to lime a lake or a watershed, they add CaCO_3 or Ca(OH)_2 to the lake to increase the pH.

(a) Why would adding CaCO_3 help to increase a lake’s pH and remediate acid precipitation problems?

3 points

Anything here about carbonates being a source of alkalinity / buffering potential was fine. Adding CaCO_3 will increase the carbonate concentration, increase the alkalinity, and increase the buffering capacity of the lake, moderating decreases in pH due to acid deposition.

(b) You are hired by a lake association that is worried about acid precipitation. Their lake currently has a pH of 5.7, but had a pH of 6.8 historically. If the pH is reduced much further, the lake will become inhospitable to valued sports fish. First, the lake association wants to know what the alkalinity of their water is.

You remember that alkalinity is determined by titration of buffering capacity with an acid. You titrate a 200 mL lake water sample with 0.1 N HCl (N=Normality) and find that it takes 2.3 mL of acid to convert all of the carbonate species to CO₂ and neutralize all and OH⁻ and anions of weak acids.

(1) What is the alkalinity of the water in units of mequivalents per liter?

4 points

As discussed in class, alkalinity is measured by titrating away the buffering capacity with an acid. You know the volume of acid used to titrate away all of the carbonates and OH⁻ (the constituents of alkalinity) from a known volume of lake water. From this you can calculate the number of mequivalents of acid used, which is equal to the mequivalents of alkalinity in the water. 1 normal = 1 equivalent/L (1 mole of charge) = 1mequiv/mL

Alkalinity = (2.3 mL) (0.1 equiv/L for HCl) (1L/1000mL) = 2.3*10⁻⁴ equivalents to use up the alkalinity in 200 mL of water. So, to figure out how many equivalents of alkalinity were in 1 liter of water, you would need to multiply by 5, because there is 5 times as much water in 1 liter as in 200 mL. So 2.3*10⁻⁴ equiv * (5/L) = 1.15*10⁻³ equivalents/L*1000mequiv/1equiv=**1.15 mequiv/L**

(2) If you assume that all of the alkalinity in the lake is due to inorganic carbon and OH⁻, what were the concentrations of CO₂, HCO₃⁻ and CO₃²⁻ in the sample before titration?

6 points

You know the pH initially, so you know [H⁺] and [OH⁻]; [H⁺]=10^{-5.7} (by definition at pH=5.7 and [OH⁻] = 10^{-8.3})

You know from the alkalinity from part 1, so you know that

$$\text{ALK} = [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}] + [\text{OH}^-] - [\text{H}^+] = 1.15 \text{ mequivalents /L.}$$

Rearranging,

$$[\text{HCO}_3^-] = \text{ALK} - 2[\text{CO}_3^{2-}] - [\text{OH}^-] + [\text{H}^+]$$

Because the pH is given as 5.7, you know that there is essentially no CO₃²⁻ (Wetzel figure 11-1 or table 11-1) and >99.999% of the carbonates are in the form of HCO₃⁻,

So you can substitute 0 = [CO₃²⁻] and calculate HCO₃⁻

$$[\text{HCO}_3^-] = 1.15 \text{ mequiv/L (alk)} - 2[\text{CO}_3^{2-}] - [\text{OH}^-] + [\text{H}^+] = 1.15 - 0 - 10^{-8.3} + 10^{-5.7} = 1.150002 \text{ mequiv/L (so basically ALL of the alkalinity is from bicarbonate – it is 1.15 mequiv/L)}$$

Since HCO_3^- has a single – charge, then 1 mequivalent/L of HCO_3^- is equal to 1 mmole/L=1mM. So $[\text{HCO}_3^-] = 1.15 \text{ mM} = 1.15 \times 10^{-3} \text{ M} = 10^{-2.94} \text{ M}$

Again, you also know the H^+ concentration – if the pH is 5.7, then the $[\text{H}^+] = 10^{-5.7} \text{ M}$.

You can then calculate CO_2 aqueous with the following equation:

$$K_1 = 10^{-6.3} = \frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{CO}_{2(\text{aq})}][\text{H}_2\text{O}]}$$

By definition $[\text{H}_2\text{O}]$ is 1.

From above, $[\text{H}^+] = 10^{-5.7} \text{ M}$

From above, $[\text{HCO}_3^-] = 1.15 \times 10^{-3} \text{ M} = 10^{-2.94} \text{ M}$

So, $10^{-6.3} = (10^{-5.7})(10^{-2.94} \text{ M})/[\text{CO}_2]$

Rearranging,

$$[\text{CO}_2] = (10^{-5.7})(10^{-2.94} \text{ M}) / (10^{-6.3}) = 10^{-8.64} / 10^{-6.3} = 10^{-2.34} \text{ M} = 4.57$$

$\times 10^{-3} \text{ M}$

$$= 4.57 \text{ mmol/L} = 4.57 \text{ mM}$$

If you used the proportions in Table 1-1 to estimate $[\text{CO}_2]$ you should have gotten approximately the correct answer:

At pH ~ 6, $\text{CO}_2 = 0.725$ of total DIC and $\text{HCO}_3^- = 0.275$, so

$$1.15 \text{ mM HCO}_3^- / x \text{ mM CO}_2 = 0.275 / 0.725$$

$$\text{so, } 0.275 x = 1.15 * 0.725$$

$$\text{so, } x = 3.03 \text{ mM CO}_2$$

So:

$[\text{H}^+] = 10^{-5.7} \text{ M}$

$[\text{HCO}_3^-] = 10^{-2.94} \text{ M}$ or **1.15 mM**

$[\text{CO}_2] = 10^{-2.34} \text{ M}$ or **4.57 mM**

and $[\text{CO}_3^{2-}] \sim 0$

If you used table 11-1 or figure 11-1 to estimate these values you did receive full credit.

(c) The association is interested in liming the lake annually to counter future additions of acidity and prevent further declines in pH. They have learned from United States Geological Survey data that the average daily rainfall brings 3,500 μmol of H^+ per m^2 of lake surface (including watershed input). The lake itself is 500,000 m^2 in surface area (5,000,000 m^3 in volume).

(1) How much CaCO_3 will the lake association need to add to their lake *annually* to counter this acidity? (you can report your answer either in equivalents or grams)

4 points

The total loading of H^+ to the lake can be found by multiplying the daily loading for the whole lake surface area and 365 days of the year. Thus the total H^+ entering the lake that needs to be buffered is:

$$3,500 \text{ umol/m}^2 \text{ day} * (500,000 \text{ m}^2) * 365 \text{ days /yr} = 6.387 \times 10^{11} \text{ umol H}^+/\text{yr} \\ = \mathbf{6.387 \times 10^{11} \text{ uequivalents /yr}} \text{ (because 1 mol H}^+ = 1 \text{ equiv H}^+)$$

Each mol of CaCO_3 has equals 2 equivalents (2 charges on the carbonate), thus you will need half of $6.387 \times 10^{11} = \mathbf{3.194 \times 10^{11} \text{ umol CaCO}_3 / \text{yr} = 319,400 \text{ mol / yr}}$

This is fine for full credit, or you could have continued to convert to weight (which you will need to do for the last problem).

The molecular weight of CaCO_3 is 100.088 g/mol, so you will need $\sim 32,000,000 \text{ g CaCO}_3$ per year, or 32,000 kg/year.

(2) If CaCO_3 costs \$25 per ton (1 ton=2000 pounds; don't forget to convert to metric), how much will it cost annually to purchase lime to buffer this small lake? (*hint*: you will need to convert your answer from the previous question from equivalents to weight to complete this if you haven't done so already)

2 points

1 pound = 0.453502 kg, so you needed 70,490 pounds CaCO_3 .

1 ton = 2,000 pounds, so you need 35.2 tons

Since 1 ton costs \$25, it would cost $25 * 35.2 = \$880$ to lime the lake (just for the CaCO_3)

The values in this table are water densities. To find the water density at 4.7 degrees, for example, you would go down the first column to find 4 degrees, and then move columns over to the right until you were at the 0.7 degree column. That value would be the water density at 4.7 degrees in units of g/cm^3 . Water density values for integer temperatures are therefore in the first column of density values.

TABLE 7. Density of water as a function of temperature from 0° C. to 35° C. at a pressure of 1 atm.

°C.	0.0	0.1	0.2	0.3	0.4
0	0.9998679	0.9998746	0.9998811	0.9998874	0.9998935
1	0.9999267	0.9999315	0.9999363	0.9999408	0.9999452
2	0.9999679	0.9999711	0.9999741	0.9999769	0.9999796
3	0.9999922	0.9999937	0.9999951	0.9999962	0.9999973
4	1.0000000	0.9999999	0.9999996	0.9999992	0.9999986
5	0.9999919	0.9999902	0.9999883	0.9999864	0.9999842
6	0.9999681	0.9999649	0.9999616	0.9999581	0.9999544
7	0.9999295	0.9999248	0.9999200	0.9999150	0.9999099
8	0.9998762	0.9998701	0.9998638	0.9998574	0.9998509
9	0.9998088	0.9998015	0.9997936	0.9997859	0.9997780
10	0.9997277	0.9997189	0.9997099	0.9997008	0.9996915
11	0.9996328	0.9996225	0.9996121	0.9996017	0.9995911
12	0.9995247	0.9995132	0.9995016	0.9994898	0.9994780
13	0.9994040	0.9993913	0.9993784	0.9993655	0.9993524
14	0.9992712	0.9992572	0.9992432	0.9992290	0.9992147
15	0.9991265	0.9991115	0.9990961	0.9990808	0.9990653
16	0.9989701	0.9989538	0.9989374	0.9989209	0.9989043
17	0.9988022	0.9987848	0.9987675	0.9987497	0.9987319
18	0.9986232	0.9986046	0.9985861	0.9985673	0.9985485
19	0.9984331	0.9984136	0.9983938	0.9983740	0.9983541
20	0.9982323	0.9982117	0.9981909	0.9981701	0.9981490
21	0.9980210	0.9979993	0.9979775	0.9979556	0.9979335
22	0.9977993	0.9977765	0.9977537	0.9977308	0.9977077
23	0.9975674	0.9975437	0.9975198	0.9974959	0.9974718
24	0.9973256	0.9973009	0.9972760	0.9972511	0.9972261
25	0.9970739	0.9970482	0.9970225	0.9969966	0.9969706
26	0.9968128	0.9967861	0.9967594	0.9967326	0.9967057
27	0.9965421	0.9965146	0.9964869	0.9964591	0.9964313
28	0.9962623	0.9962338	0.9962052	0.9961766	0.9961478
29	0.9959725	0.9959440	0.9959146	0.9958850	0.9958554
30	0.9956726	0.9956434	0.9956131	0.9955826	0.9955521
31	0.9953692	0.9953380	0.9953068	0.9952755	0.9952442
32	0.9950542	0.9950222	0.9949901	0.9949580	0.9949258
33	0.9947308	0.9946980	0.9946651	0.9946321	0.9945991
34	0.9943991	0.9943655	0.9943319	0.9942981	0.9942644
35	0.9940394	0.9940025	0.9939656	0.9939286	0.9938914

TABLE 7 (Continued)

°C.	0.5	0.6	0.7	0.8	0.9
0	0.9998095	0.9998053	0.9998010	0.9998010	0.9998016
1	0.9998404	0.9998354	0.9998303	0.9998310	0.9998315
2	0.9998621	0.9998564	0.9998506	0.9998507	0.9998505
3	0.9998761	0.9998698	0.9998634	0.9998634	1.0000000
4	0.9998879	0.9998810	0.9998736	0.9998737	0.9998734
5	0.99989819	0.9998915	0.9998839	0.9998841	0.9998842
6	0.9999026	0.9998967	0.9998886	0.9998884	0.9998880
7	0.9999046	0.9998992	0.9998936	0.9998936	0.9998932
8	0.9999042	0.9998974	0.9998930	0.9998930	0.9998926
9	0.9999029	0.9998957	0.9998913	0.9998913	0.9998909
10	0.99989830	0.9998924	0.9998880	0.9998880	0.9998876
11	0.9998903	0.9998864	0.9998820	0.9998820	0.9998816
12	0.9998790	0.9998753	0.9998710	0.9998710	0.9998706
13	0.9998644	0.9998608	0.9998573	0.9998573	0.9998569
14	0.9998468	0.9998433	0.9998400	0.9998400	0.9998396
15	0.9998263	0.9998229	0.9998200	0.9998200	0.9998196
16	0.9998029	0.9998000	0.9997975	0.9997975	0.9997971
17	0.9997766	0.9997742	0.9997720	0.9997720	0.9997716
18	0.9997475	0.9997453	0.9997435	0.9997435	0.9997431
19	0.9997157	0.9997137	0.9997120	0.9997120	0.9997116
20	0.9996813	0.9996795	0.9996780	0.9996780	0.9996776
21	0.9996344	0.9996328	0.9996315	0.9996315	0.9996311
22	0.9995850	0.9995836	0.9995825	0.9995825	0.9995821
23	0.9995332	0.9995319	0.9995309	0.9995309	0.9995305
24	0.9994790	0.9994778	0.9994769	0.9994769	0.9994765
25	0.9994224	0.9994213	0.9994205	0.9994205	0.9994201
26	0.9993635	0.9993625	0.9993617	0.9993617	0.9993613
27	0.9993023	0.9993014	0.9993006	0.9993006	0.9993002
28	0.9992388	0.9992380	0.9992372	0.9992372	0.9992368
29	0.9991731	0.9991723	0.9991715	0.9991715	0.9991711
30	0.9991052	0.9991045	0.9991037	0.9991037	0.9991033
31	0.9990352	0.9990345	0.9990337	0.9990337	0.9990333
32	0.9989631	0.9989624	0.9989616	0.9989616	0.9989612
33	0.9988890	0.9988883	0.9988875	0.9988875	0.9988871
34	0.9988129	0.9988122	0.9988114	0.9988114	0.9988110
35	0.9987348	0.9987341	0.9987333	0.9987333	0.9987329