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The baseflow correlation method with multiple gauged sites

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KEYWORDS
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Summary Low streamflow estimates are required for water quality and quantity management purposes. The present research investigates the baseflow correlation method using information from multiple gauged sites to estimate low flow statistics at a partial record site. The baseflow correlation method is an information transfer technique that is used to estimate low streamflow statistics at a partial record site by correlating a limited number of measured streamflow discharges during baseflow conditions at the partial record site with those at nearby-gauged sites. Traditionally, the baseflow correlation method has been employed using a single gauged site to transfer information to the partial record site. Seven new estimators using information from multiple gauged sites are proposed and examined in this research, and compared to the baseflow correlation method with a single gauged site. A delete-one cross-validation is performed to assess the baseflow correlation estimators by employing daily streamflow values at more than 1300 USGS HCDN gauged sites. A comparison of using multiple gauged sites with using a single site shows that the performance can be improved by using multiple site information, especially when less than 10 baseflow measurements are used.

Introduction

Developing accurate estimates of low streamflows is crucial for effective water resources planning and management (Smakhtin, 2001). The 7-day, 10-year low flow ($Q_{7,10}$) and 7-day, 2-year low flow ($Q_{7,2}$) are commonly used low flow statistics in the United States (Riggs, 1980). The $Q_{7,10}$ is the annual 7-day minimum flow that is expected to be exceeded on average in 9 out of every 10 years, which is equivalent to the 10th percentile of the distribution of 7-day annual minimum streamflows (Reilly and Kroll, 2003). The return period for the $Q_{7,10}$ is 10 years, by definition. The $Q_{7,2}$ is the 50th percentile of the distribution of 7-day annual minimum streamflows and its return period is 2 years.

A frequency analysis can be employed to estimate low streamflow statistics when historic streamflow records are

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available at the streamflow site of interest (Riggs, 1972; Stedinger et al., 1993). When no historic record is available, reliable methods for estimating streamflow statistics are still needed. Regional regression is a common method used for estimating low flow statistics at ungauged river sites (Smakhtin, 2001; Kroll et al., 2004). While there are some examples of well performing regression models for low flows (Gustard et al., 1992; Aschwanden and Kan, 1999; Laaha and Blöschl, 2006; Engeland et al., 2006; Ries, 2002), this method sometimes performs poorly in practice for estimating low flow statistics (Thomas and Benson, 1970; Barnes, 1985; Hammett, 1985; Arihood and Glatfelter, 1986; Vogel and Kroll, 1992; Ries, 1994; Kroll et al., 2004). The baseflow correlation method is an alternative method if a number of baseflow measurements are available at the site of interest (the site is referred to as partial record site hereafter). This method correlates a nominal number of baseflow measurements obtained at the partial record site with concurrent daily flows at a nearby-gauged site to estimate low flow statistics at the partial record site (Riggs, 1965, 1972).

Stedinger and Thomas (1985) examined the baseflow correlation method using 20 pairs of stream sites and found it to be the best estimator among the five estimators investigated in their study. Reilly and Kroll (2003) examined the baseflow correlation method at more than 1300 river sites of the United States Geological Survey's (USGS) Hydro-Climatic Data Network (HCDN) (Slack and Landwehr, 1992). Zhang and Kroll (2007) revisited Reilly and Kroll's analysis, investigated more explanatory parameters and model assumptions and provided guidance on how to implement the baseflow correlation method in practice. Reilly and Kroll (2003) and Zhang and Kroll (2007) found that the baseflow correlation method performs well in the United States when baseflow measurements are obtained by randomly choosing one baseflow measurement from consecutive recessions (referred to as the ''recession'' method) and the potential candidate gauged sites are restricted within 200 km. They also suggested using at least 10 baseflow measurements. Zhang and Kroll (2007) concluded that the baseflow measurements at the partial record site should be obtained during the low flow season and as far as possible from runoff events. Zhang and Kroll also found that the basic assumptions of the baseflow correlation method appear reasonable in their study regions.

All of the aforementioned investigations of the baseflow correlation method employed one nearby-gauged site and did not use information from multiple nearby-gauged sites. However, it is common to find multiple nearby-gauged sites with baseflow conditions on the same days as those at which baseflow is measured at the partial record site. Laaha and Blöschl (2005) investigated the record argumentation method for estimating low flow statistics when multiple gauged sites (referred to as donor sites in Laaha and Blöschl, 2005) are available. To our knowledge, how to best use the information of multiple nearby-gauged sites in the baseflow correlation method has never been investigated. This study proposes seven new estimators to utilize information from multiple nearby-gauged sites and compares them with the baseflow correlation method using a single gauged site. The goal of this study is to examine if the performance of the baseflow correlation method could be improved by using information from multiple nearby-gauged sites. To compare the methods, the $Q_{7,10}$ and $Q_{7,2}$ are estimated with multiple gauged site estimators and the single site estimator using the HCDN dataset.

Methodology

The baseflow correlation method is an information transfer technique that employs a small quantity of streamflow data at a site to estimate low streamflow statistics. The method has several basic principles and assumptions that are summarized below. It assumes a linear relationship between the logarithm of the annual minimum d-day flows at the partial record site and the logarithm of the annual minimum d-day flows at the gauged site. It further assumes that the relationship between annual minimum d-day flows is similar to the relationship between instantaneous baseflows. Zhang and Kroll (2007) investigated this assumption and found it to be reasonable for 7-day annual minimum flows in the United States. The linear relationship is adapted to correlate the logarithm of the baseflow measurements at the partial record site with the logarithm of the corresponding baseflows at the gauged site. The log-space mean and variance of the annual minimum d-day streamflows are then estimated based on the linear relationship. It is also assumed that the annual minimum streamflows are well described by a log-Pearson type 3 (LP3) distribution. The LP3 distribution has been employed by the USGS for describing annual minimum streamflow series in the United States (Rumenick and Grubbs, 1996; Wandle and Randall, 1993; Barnes, 1985; Hortness, 2006). The last assumption is that the frequency factor for the partial record site, K_y, is assumed equal to the frequency factor for the gauged site, K_x. Low streamflow statistics are then estimated based on the above principles. For further details about the baseflow correlation method with a single gauged site see Stedinger and Thomas (1985) and Zhang and Kroll (2007).

This study proposes seven new baseflow correlation estimators using information from multiple nearby-gauged sites. The first four estimators combine the estimates using a single gauged site into a single adjusted value using averaging techniques. The four averaging techniques examined include the average in log-space ($\widehat{Q}_{M1}$), the average in real-space ($\widehat{Q}_{M2}$), the variance-weighted average in log-space ($\widehat{Q}_{M3}$), and the variance-weighted average in real-space ($\widehat{Q}_{M4}$). As the real-space estimators are very similar to the log-space estimators, only the log-space estimators are shown below:

$$\widehat{Q}_{M1} = \exp\left(\sum_{i=1}^{N} \ln(\widehat{Q}_{7,Ti})/N\right) \tag{1}$$

$$\widehat{Q}_{M3} = \exp\left(\frac{\sum_{i=1}^{N} \ln(\widehat{Q}_{7,Ti})/\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))}{\sum_{i=1}^{N} 1/\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))}\right) \tag{2}$$

where $\widehat{Q}_{7,Ti}$ is the ith estimator of $Q_{7,T}$ using the ith candidate long record site, and N is the number of candidate gauged sites. Stedinger and Thomas (1985) provided an approximation of the log-space variance $\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))$. The variance term $\mathrm{Var}(\widehat{Q}_{7,Ti})$ can be derived using a first-order Taylor series expansion as

$$\mathrm{Var}(\widehat{Q}_{7,Ti}) = (\widehat{Q}_{7,Ti})^2 \mathrm{Var}(\ln(\widehat{Q}_{7,Ti})) \tag{3}$$

Another estimator could be developed by multiple linear regression, where the log-space baseflow measurements from the partial record sites are regressed against the log-space baseflows from the multiple gauged sites. The problem with this technique is the baseflows from gauged sites in a region are typically highly correlated, which can cause multicollinearity in the regression analysis. The presence of multicollinearity can result in: (a) large changes in the estimated parameters when a variable is added or deleted, (b) large changes in the parameters when a data point is altered or dropped, (c) the estimated parameters having the wrong sign or questionable magnitude, (d) parameter estimators with large standard errors, and (e) parameter estimators that are correlated (Neter et al., 1989; Johnston, 1972; Luz, 2005).

To overcome this potential problem, the fifth estimator proposed in this study employs a principle component regression technique. In this analysis we create a fictitious streamflow site, referred to here as a ''virtual site'', whose flows are a linear combination of the flows from the other gauged sites. A virtual gauged site is constructed using the 7-day annual minimums and baseflow measurements at multiple near-by gauged sites. The log-space 7-day annual minimums at the virtual gauged site, x_{iv1}, are assumed to be the first principle component of the annual d-day minimums of the N gauged sites. It is computed as

$$x_{iv1} = e_{11}x_{i1} + e_{12}x_{i2} + \cdots + e_{1N}x_{iN} \tag{4}$$

where $(e_{11}, e_{12} \ldots e_{1N})$ is the first eigenvector of the covariance matrix of the 7-day annual minimums at the N gauged sites, and $x_{i1}, x_{i2} \ldots x_{iN}$ are the log-space 7-day annual minimums at the N gauged sites. Only 7-day annual minimums with the same concurrent time period across all gauged sites are employed to derive estimators of e_{1i}, $\hat{e}_{1i}$. The baseflow measurements in log-space at the virtual gauged site, $\tilde{x}_{iv1}$, are calculated using the weights obtained in Eq. (4) as

$$\tilde{x}_{iv1} = \hat{e}_{11}\tilde{x}_{i1} + \hat{e}_{12}\tilde{x}_{i2} + \cdots + \hat{e}_{1N}\tilde{x}_{iN} \tag{5}$$

where $\tilde{x}_{i1}, \tilde{x}_{i2} \ldots \tilde{x}_{iN}$ are the concurrent baseflow measurements in log-space at the N gauged sites. The baseflow correlation method is performed using the virtual gauged site as the gauged site in the analysis. The log-space baseflow measurements at the partial record site are regressed against the log-space baseflows at the virtual site as

$$\tilde{y}_i = \alpha + \beta\tilde{x}_{iv1} + \varepsilon \tag{6}$$

The log-space mean and variance, $\hat{\mu}_y$ and $\hat{\sigma}_y$, are then estimated by

$$\hat{\mu}_y = a + bm_{xv1} \tag{7}$$

$$\hat{\sigma}_y^2 = b^2 s_{xv1}^2 + s_e^2\left(1 - \frac{s_{xv1}^2}{(L-1)s_{\tilde{x}v1}^2}\right) \tag{8}$$

where m_{xv1} is the log-space mean of the 7-day annual minimum flows at the virtual gauged site, s_{xv1}^2 is the log-space variance of the 7-day annual minimum flows at the virtual gauged site, $s_{\tilde{x}v1}^2$ is the sample variance of the logarithms of the baseflows at the virtual gauged site, and a, b, and s_e^2 are OLS estimators of the parameters of the log-linear relationship (Draper and Smith, 1966). The estimator of the $Q_{7,T}$ is then calculated as

$$\ln(\widehat{Q}_{M5}) = \hat{\mu}_y + K_y\hat{\sigma}_y \tag{9}$$

The frequency factor at the partial record site, K_y, is assumed equal to the frequency factor for the 7-day annual minimum series at the virtual gauged site.

The last two estimators, $\widehat{Q}_{M6}$ and $\widehat{Q}_{M7}$, employ information from multiple sites to estimate the log-space mean and variance by combining the log-space means and variances estimated with a single gauged site. For $\widehat{Q}_{M6}$, $\hat{\mu}_{M6}$ and $\hat{\sigma}_{M6}^2$ are estimated by

$$\hat{\mu}_{M6} = \sum_{i=1}^{N} \hat{\mu}_i \Big/ N \tag{10}$$

$$\hat{\sigma}_{M6}^2 = \sum_{i=1}^{N} \hat{\sigma}_i^2 \Big/ N \tag{11}$$

where $\hat{\mu}_i$ and $\hat{\sigma}_i^2$ are the log-space mean and variance estimators using the ith candidate gauged site, respectively, and N is the number of candidate gauged sites. $\widehat{Q}_{M6}$ is then estimated by

$$\ln\left(\widehat{Q}_{M6}\right) = \hat{\mu}_{M6} + K_{M6}\hat{\sigma}_{M6} \tag{12}$$

where K_{M6} is the average of the frequency factors of the multiple gauged sites. For the last estimator $\widehat{Q}_{M7}$, $\hat{\mu}_{M7}$ and $\hat{\sigma}_{M7}^2$ are estimated by

$$\hat{\mu}_{M7} = \frac{\sum_{i=1}^{N}(\hat{\mu}_i/\mathrm{var}(\hat{\mu}_i))}{\sum_{i=1}^{N} 1/\mathrm{var}(\hat{\mu}_i)} \tag{13}$$

$$\hat{\sigma}_{M7}^2 = \frac{\sum_{i=1}^{N}(\hat{\sigma}_i^2/\mathrm{var}(\hat{\sigma}_i^2))}{\sum_{i=1}^{N}(1/\mathrm{var}(\hat{\sigma}_i^2))} \tag{14}$$

where var $(\hat{\mu}_i)$ and var $(\hat{\sigma}_i^2)$ are the variance of log-space mean and variance estimators using the ith candidate gauged site, respectively. Stedinger and Thomas (1985) provided approximations of var $(\hat{\mu}_i)$ and var $(\hat{\sigma}_i^2)$. $\widehat{Q}_{M7}$ is then estimated by

$$\ln(\widehat{Q}_{M7}) = \hat{\mu}_{M7} + K_{M7}\hat{\sigma}_{M7} \tag{15}$$

where K_{M7} is the average of the frequency factors of the multiple gauged sites.

Data

The USGS's Hydro-Climatic Data Network (HCDN) (Slack and Landwehr, 1992) is used in this study. The HCDN consists of approximately 1600 stream gauging stations located throughout the United States and its Territories. The data set includes streamflow records, as well as variables describing the topography, climate, and geology of each watershed. This dataset was developed specifically for examining the effects of climate change on hydrologic conditions, and has been employed in many streamflow studies (for example see Vogel et al., 1997; Douglas et al., 2000; Kroll et al., 2004; Reilly and Kroll, 2003; Zhang and Kroll, 2007). Record lengths vary from 20 to 114 years of daily average streamflow measurements. This experiment employs only HCDN stream gauge stations that are designated as having data suitable at a daily time step, where the at-site estimate of the $Q_{7,10}$ or $Q_{7,2}$ is greater than zero.

Experiment

A delete-one cross-validation analysis is employed to evaluate the baseflow correlation method with multiple gauged sites for estimating low streamflow statistics. Note that a delete-one cross-validation is often referred to as a jackknife simulation, while distinction is sometimes made; cross-validation is used to estimate generalization error, while the jackknife is typically used to estimate the bias and standard error of a statistics (Efron, 1982).

The results of this analysis are compared with the results of the baseflow correlation analysis with a single gauged site (Zhang and Kroll, 2007). The analysis with multiple gauged sites is as follows:

(1) Designate one HCDN site as the hypothetical partial record site and the remainder of the sites as potential gauged sites.
(2) Select one baseflow segment at the hypothetical partial record site using the recession method. The recession method selects one random baseflow discharge from consecutive recessions, starting at a random starting recession (Reilly and Kroll, 2003). A baseflow segment is a series of baseflow measurements at the partial record site. The number of baseflow measurements in a baseflow segment is called the segment length. Segment lengths of 5, 10, and 15 days are used in this study. Streamflow is designated as baseflow if it occurs after at least 5 days of continuously decreasing streamflow. In addition, only flows from July through October are used, which is generally the low flow period in the United States. Zhang and Kroll (2007) showed that the baseflow correlation method works much better during this period as opposed to across the entire year.
(3) Find the candidate gauged sites that have baseflow conditions on the same days as those comprising the baseflow segment at the partial record site. The gauged sites are restricted within 200 km of the partial record site, as suggested by Reilly and Kroll (2003). If there is not more than one candidate gauged site, this baseflow segment is discarded.
(4) If there is more than one candidate gauged site, calculate $Q_{7,T}$ estimates employing a single candidate gauged site for each of the candidate sites and compute the correlation coefficients between the baseflows at each candidate gauged site and the partial record site.
(5) If there is no candidate gauged site that has a correlation coefficient greater than or equal to 0.6, the baseflow segment is discarded, and step 2 is reperformed.
(6) If there is at least 2 candidate gauged sites, and the correlation coefficient between baseflows from at least one gauged site and the partial record site are greater than 0.6, the baseflow segment is employed as follows:
   (i) For the single site estimator, the baseflow correlation estimator is taken with the gauged site that produces the smallest variance for the baseflow correlation estimator.
   (ii) For the four multiple gauged site estimators based on averaging, low streamflow statistics are estimated based on the averaging techniques.
   (iii) For the principle component analysis estimators, the eigenvector of the variance–covariance matrix for concurrent 7-day annual minimums at the gauged sites is calculated. Then the first principle components are calculated with the eigenvector and the concurrent daily baseflows at the gauged sites. The $Q_{7,T}$ estimator $\widehat{Q}_{M5}$ is computed using the first principle component and the corresponding baseflow measurements at the partial record site.
   (iv) The log-space mean and variance are estimated by combining the means and variances estimated using each gauged site and then the last two multiple site estimators are estimated via Eqs. (12) and (15),
(7) Repeat the process for all baseflow segments at the partial record site. The number of baseflow segments constructed is equal to the total number of baseflow days divided by the segment length.
(8) Repeat the above seven steps for all sites by sequentially designating each site as the partial record site.
(9) Compute summary statistics by comparing the computed $Q_{7,T}$ to at-site $Q_{7,T}$ at the partial record site. The summary statistics are then used to assess the numerous estimators.

The unit area absolute difference (UAAD) is used as a performance metric in this analysis. The UAAD for each partial record site is computed as

$$\mathrm{UAAD} = \frac{\sum_{i=1}^{M}\left(\frac{|\widehat{Q}_{7,Ti} - Q_{7,T}|}{A}\right)}{M} \tag{16}$$

where $\widehat{Q}_{7,Ti}$ is the ith estimator of the $Q_{7,T}$ at the partial record site, $Q_{7,T}$ is the ''true value'' of the $Q_{7,T}$ at the partial record site, A is the drainage area, and M is the number of baseflow segments from which estimates of the $Q_{7,T}$ have been obtained. $Q_{7,T}$ is the at-site estimator obtained using the entire historic record, fitting a LP3 distribution by method of moments (Stedinger et al., 1993), and estimating the 10th percentile of the distribution for $Q_{7,10}$ and the 50th percentile for $Q_{7,2}$. The units of UAAD is cfs per square mile. The UAAD appears to be best suited for comparing low streamflow statistic estimators across sites with a large range of low flow magnitudes (Zhang and Kroll, 2007). The results for all the partial record sites across the United States are computed as the average of each performance measure, weighted by the record length (in years) of the site:

$$\text{Average UAAD} = \frac{\sum_{i=1}^{G} \mathrm{UAAD}_i * \text{Record Length}_i}{\sum_{i=1}^{G} \text{Record Length}_i} \tag{17}$$

where G is the number of partial record sites over the conterminous United States.

To compare the relative performance of each estimator, a performance ratio is employed (Kroll and Stedinger, 1996). The performance ratio for the estimators is computed as

$$\mathrm{PR}_j = \frac{\mathrm{UAAD}_{\mathrm{Best}}}{\mathrm{UAAD}_j} \tag{18}$$

where $UAAD_{Best}$ is the UAAD of the best estimator which has the smallest UAAD, and $UAAD_j$ is the UAAD of the estimator of interest. The performance ratio will range from 0 to 1 for all estimators, and be equal to 1 for the best estimator.

As the Root Mean Square Error (RMSE) is usually an unbiased estimate of the expected error and widely used in the hydrologic literature, it is also estimated in this study to facilitate the comparison of the examined estimators. Laaha and Blöschl (2005) employed RMSE of specific low flow discharges, to investigate the performance of the record argumentation method for estimating low flow statistics. Here we employ the Root Unit Area MSE (RUAMSE) that is calculated by

$$\mathrm{RUAMSE} = \sqrt{\frac{\sum_{i=1}^{M}\left(\frac{\widehat{Q}_{7,Ti}-Q_{7,T}}{A}\right)^2}{M}} \tag{19}$$

Results

Results for estimating $Q_{7,10}$

The results of this experiment for estimating $Q_{7,10}$ as well as a comparison with Zhang and Kroll's (2007) single site estimator are presented here. The PR for $Q_{7,10}$ estimators over the conterminous United States is shown in Fig. 1. The multiple gauged site estimators based on averaging single site estimators ($\widehat{Q}_{M1}$, $\widehat{Q}_{M2}$, $\widehat{Q}_{M3}$, and $\widehat{Q}_{M4}$) are typically better than the estimator using a single site ($\widehat{Q}_{SIN}$), but the improvement decreases as the segment length increases. For a segment length of 5 days, the performance ratio of $\widehat{Q}_{SIN}$ is only 0.81, while for a segment length of 10 days the performance ratio is 0.91, and for 15 days it is 0.98. In addition, the multiple gauged site estimators based on averaging single site estimators ($\widehat{Q}_{M1}$, $\widehat{Q}_{M2}$, $\widehat{Q}_{M3}$, and $\widehat{Q}_{M4}$) always perform better than using the principal component-based estimator ($\widehat{Q}_{M5}$). The two multiple site moment estimators, $\widehat{Q}_{M6}$ and $\widehat{Q}_{M7}$, have a very similar performance pattern. For segment lengths of 5 and 10 days, they perform as well as the averaging estimators ($\widehat{Q}_{M1}$, $\widehat{Q}_{M2}$, $\widehat{Q}_{M3}$, and $\widehat{Q}_{M4}$), but for segment of 15 days the averaging estimators outperform $\widehat{Q}_{M6}$ and $\widehat{Q}_{M7}$.

To compare the multiple site estimators with the single site estimator in more details, the average UAAD using segment lengths of 5, 10, and 15 days for each estimator across the conterminous United States are shown in Fig. 2. It should be noted that the average UAAD across the conterminous United States in Fig. 1 is weighted on the record length (equation (17)), while the UAAD for each estimator shown in Fig. 2 is based on the unweighted UAAD values. From Fig. 2, when the segment length is 5 days the average UAAD of all the multiple gauged site estimators (ranging from 0.050–0.054 cfs/mi^2) is less than the UAAD of the single site estimator (0.062 cfs/mi^2). A hypothesis test with a null hypothesis that all average UAADs were equal was performed and rejected at the 1% significance level. When the same hypothesis test was tested excluding the single site estimator the null hypothesis could not be rejected at a 10% significance level. Using a segment length of 10 days, the mean UAAD of the single site estimator (0.044 cfs/mi^2) is slightly greater than the mean UAAD of all the multiple gauged site estimators (approximately 0.040 cfs/mi^2), though this difference was not significant at a 10% significance level. The results using a segment length of 15 days show that the average UAAD of the single site estimator performs as good as the multiple site estimators. The average RUAMSE using segment lengths of 5, 10, and 15 days for various estimators are shown in Fig. 3. It shows similar results with respect to the relative performance of various estimators. For a segment length of 5 days, the average RUAMSE decreases from 0.083 to 0.065 cfs/mi^2 if multiple site estimators are used instead of the single site estimator. For segment length of 10 days, the average RUAMSE are 0.053 and 0.047 cfs/mi^2 for single site estimator and $\widehat{Q}_{M3}$, respectively. For segment length of 15 days, the average RUAMSE of the single site estimator is almost as good as $\widehat{Q}_{M3}$.

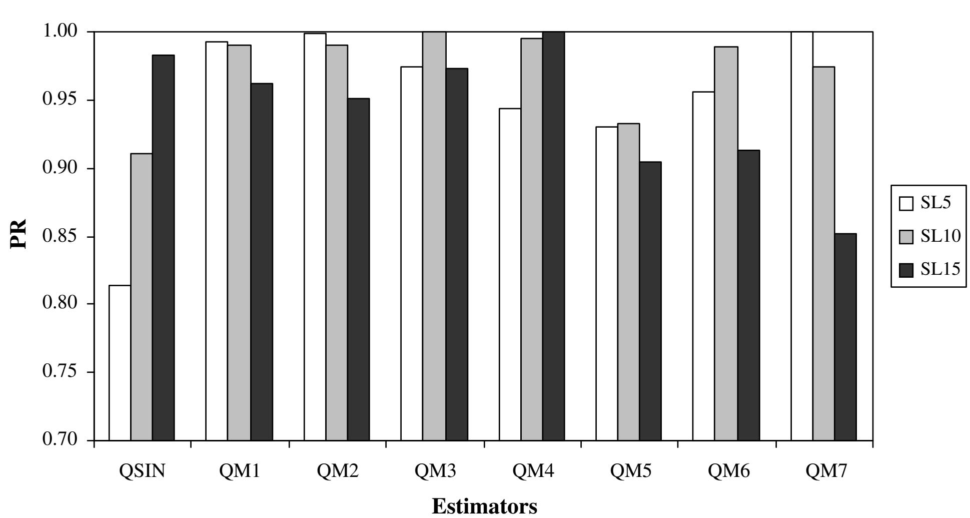

Figure 1 The performance ratio (PR) for $Q_{7,10}$ (7-day 10-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1–QM7 are the multiple site estimators.

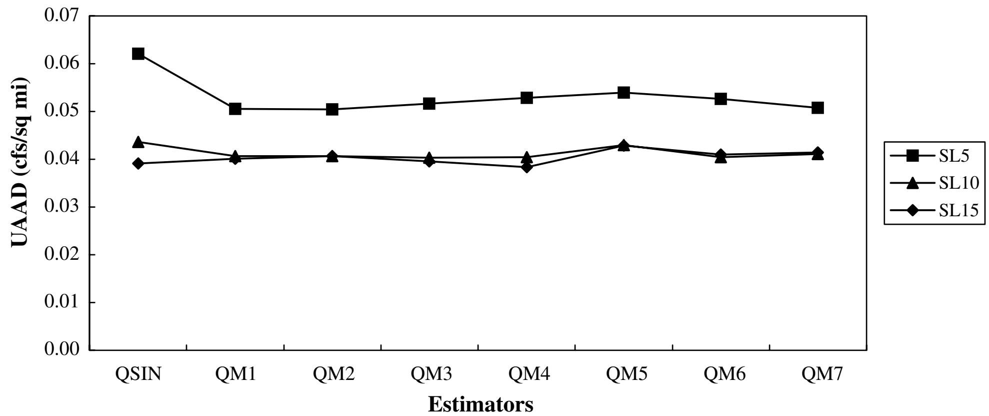

Figure 2 The Unit Area Absolute Difference (UAAD) for $Q_{7,10}$ (7-day 10-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1—QM7 are the multiple site estimators.

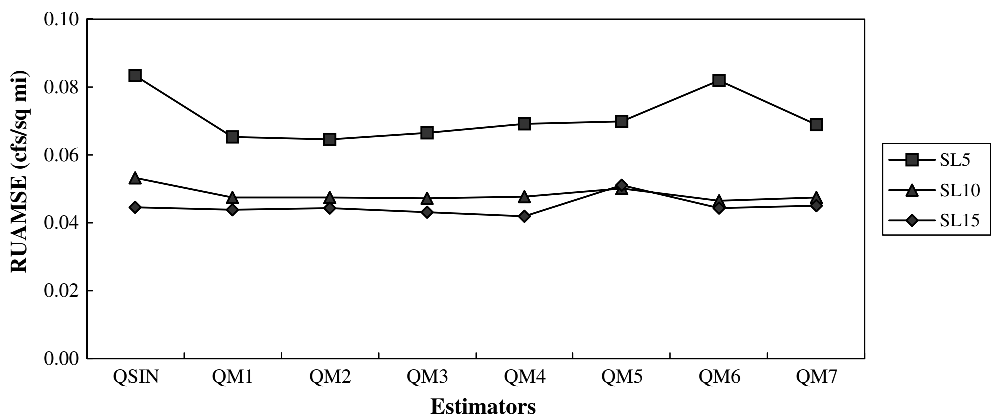

Figure 3 The Root Unit Area Mean Square Error (RUAMSE) for $Q_{7,10}$ (7-day 10-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1—QM7 are the multiple site estimators.

From the above analysis, the multiple site estimators of $Q_{7,10}$ appear to outperform the single site estimator for estimating the $Q_{7,10}$ when 10 or less baseflow measurements are available. When more than 10 baseflow measurements are available, the gain by using the multiple site estimators is negligible. Of the multiple site estimators of $Q_{7,10}$, $\widehat{Q}_{M1}$ and $\widehat{Q}_{M3}$ appear to perform best among the seven methods explored for using information of multiple nearby-gauged sites, though the performance differences among the multiple site estimators are insignificant.

Results for estimating $Q_{7,2}$

The performance ratios over the conterminous United States for the $Q_{7,2}$ estimators are shown in Fig. 4. Again for segment lengths of 5 and 10 days, the multiple site estimators perform better than the single site estimator. The performance ratios of $\widehat{Q}_{SIN}$ are 0.84 and 0.91 for segment lengths of 5 and 10 days, respectively. For a segment length of 15 days, the single site estimator performs best among all the estimators. Among the multiple site estimators, $\widehat{Q}_{M3}$ performs best as it has the best performance ratios for segment lengths of 5, 10, and 15 days. The average UAAD for each estimator using segment lengths of 5, 10, and 15 days are shown in Fig. 5. For a segment length of 5 days, the average UAAD of the multiple site estimators is less than the average UAAD of the single site estimator (significant at a 1% level). For a segment length of 10 and 15 days, the difference in the average UAAD is not significant at a 10% level. The average RUAMSE for various estimators are shown in Fig. 6. Again it shows similar results as UAAD with respect to the relative performance of various estimators. For a segment length of 5 days, the RUSMAE decreases from 0.096 for the single site estimator to 0.074 cfs/ mi^2 for $\widehat{Q}_{M3}$. The difference of RUAMSE among all the examined estimators is not significant for segment lengths of 10 or 15 days. There appears to be little difference between the multiple gauged site estimators of $Q_{7,2}$, while $\widehat{Q}_{M3}$ shows highest performance ratios on average over the simulation parameters examined.

In general, the multiple site estimators of $Q_{7,2}$ appear to outperform the single site estimator for estimating the $Q_{7,2}$ when 10 or less baseflow measurements are available. The single site estimator performs as well as the multiple site estimators when 15 baseflow measurements are available. $\widehat{Q}_{M3}$ appears to perform best among the seven multiple site $Q_{7,2}$ estimators, though the performance differences among the multiple site estimators are not significant.

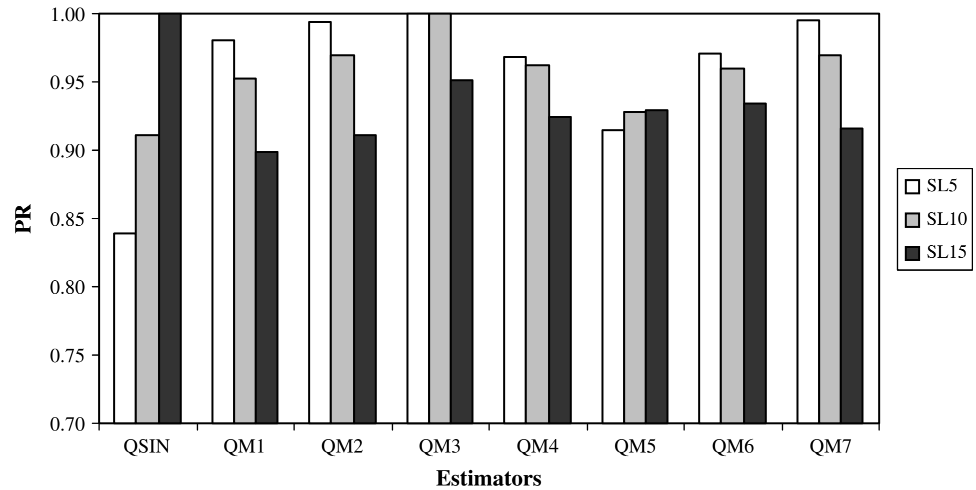

Figure 4 The Performance Ratio (PR) for $Q_{7,2}$ (7-day 2-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1—QM7 are the multiple site estimators.

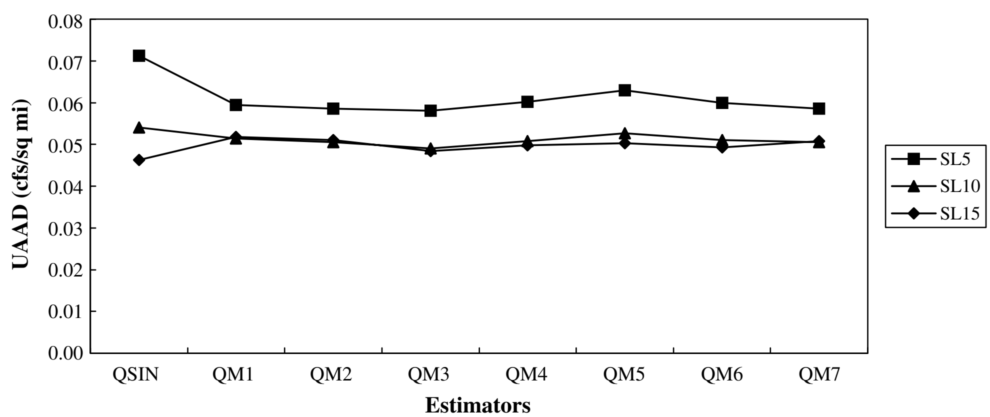

Figure 5 The Unit Area Absolute Difference (UAAD) for $Q_{7,2}$ (7-day 2-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1—QM7 are the multiple site estimators.

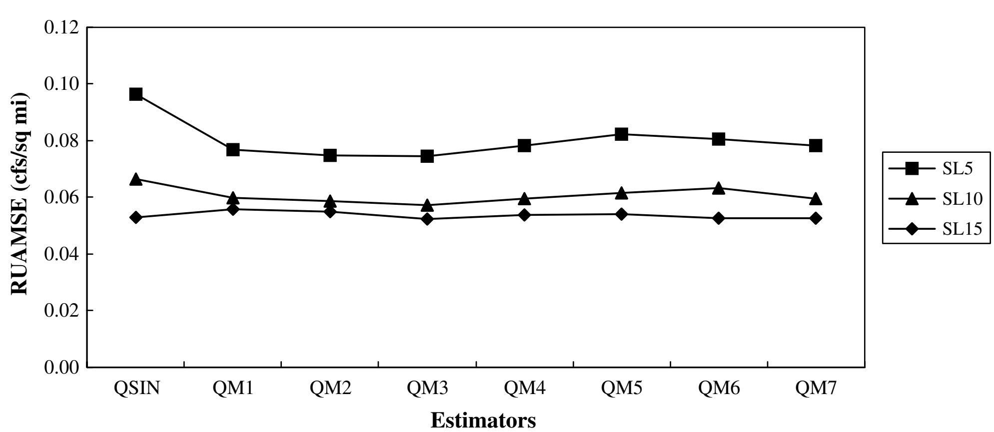

Figure 6 The Root Unit Area Mean Square Error (RUAMSE) for $Q_{7,2}$ (7-day 2-year low flow) estimators for segment lengths of 5 (SL5), 10 (SL10), and 15 (SL15) days. QSIN indicates the single site estimator, while QM1—QM7 are the multiple site estimators.

Discussions

From the above analysis, the multiple site estimators appear to improve the performance of the baseflow correlation method for estimating both $Q_{7,10}$ and $Q_{7,2}$ when less than 10 baseflow measurements are available. When 15 baseflow measurements are available, the single site estimator performs as well as the multiple gauged site estimators. The results also imply that $\widehat{Q}_{M3}$ appears to perform best among the multiple site estimators though the difference among all the multiple site estimators is not significant for segment lengths of 5, 10, or 15 days. Three factors have a large impact on the error of $Q_{7,10}$ and $Q_{7,2}$ baseflow correlation estimators: (1) the number of concurrent baseflow measurements, (2) the strength of the linear relationship between the log-space baseflow measurements, and (3) the error in the estimators of the gauged site's moments. The number of baseflow measurements and the strength of the linear relationship between the baseflows impacts the variance of the regression model residuals, and thus impacts estimators derived at the partially gauged site. The results of Stedinger and Thomas (1985) and Zhang and Kroll (2007) imply that the major factor controlling the accuracy of the baseflow correlation method is the number of baseflow measurements when the number of concurrent baseflow measurements is small, and the controlling factor is the precision of the estimators of the moments of the gauged sites when the number of concurrent baseflow measurements is larger. This appears to be the reason that the multiple site estimators outperform the single site estimator when a small number of concurrent baseflow measurements are available. The recommended multiple site estimator $\widehat{Q}_{M3}$ provides a valuable tool for estimating low streamflow statistics, especially when only a small number of baseflow measurements can be obtained in the otherwise ungauged site due to financial or time limit. The other advantage of $\widehat{Q}_{M3}$ is that it is easy to implement this estimator in practice, especially when compared with the implementation of estimators based on the principle component analysis technique ($\widehat{Q}_{M5}$). To assess the precision or sampling variability of the recommended estimator, the variance of the estimator is warranted. Zhang (2005) derived a first-order estimate of the variance of the estimator, but the complexity of the expression prohibits its use in practice. An approximation of the variance of $\widehat{Q}_{M3}$ is derived below.

From Eq. (9), the variance of $\widehat{Q}_{M3}$ can be expressed as

$$\mathrm{Var}(\ln(\widehat{Q}_{M3})) = \frac{\sum_{i=1}^{N} \frac{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))}{(\mathrm{Var}(\ln(\widehat{Q}_{7,Ti})))^2} + 2\sum_{i<j}\sum_{j=1}^{N} \frac{\mathrm{Cov}(\ln(\widehat{Q}_{7,Ti}),\ln(\widehat{Q}_{7,Ti}))}{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))\mathrm{Var}(\ln(\widehat{Q}_{7,Tj}))}}{\sum_{i=1}^{N} \frac{1}{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))^2}} \tag{20}$$

To get the variance of $\widehat{Q}_{M3}$, the covariance of $\ln(\widehat{Q}_{7,Ti})$ and $\ln(\widehat{Q}_{7,10j})$ is required. The covariance can be expressed as

$$\mathrm{Cov}(\ln(\widehat{Q}_{7,Ti}), \ln(\widehat{Q}_{7,Ti})) = \rho_{ij}\sqrt{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))\mathrm{Var}(\ln(\widehat{Q}_{7,Tj}))} \tag{21}$$

where ρ_{ij} is the Pearson correlation coefficient between $\ln(\widehat{Q}_{7,Ti})$ and $\ln(\widehat{Q}_{7,10j})$. We expect ρ_{ij} to be high, since the correlation coefficient between the baseflows at each gauged site and the partial record site is at least 0.6, and it typically greater than 0.8 in our simulations. We can thus develop an upper bound estimator of the covariance term by assuming $\rho_{ij} = 1$:

$$Cov(\ln(\widehat{Q}_{7,Ti}), \ln(\widehat{Q}_{7,Ti})) \approx \sqrt{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))\mathrm{Var}(\ln(\widehat{Q}_{7,Tj}))} \tag{22}$$

Substituting Eq. (22) into Eq. (20) we obtain an upper bound variance approximation of $\widehat{Q}_{M3}$ as

$$\mathrm{Var}(\ln(\widehat{Q}_{M3})) \leqslant \frac{\sum_{i=1}^{N} \frac{1}{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))} + 2\sum_{i<j}\sum_{j=1}^{N} \frac{1}{\sqrt{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))\mathrm{Var}(\ln(\widehat{Q}_{7,Tj}))}}}{\sum_{i=1}^{N} \frac{1}{\mathrm{Var}(\ln(\widehat{Q}_{7,Ti}))^2}} \tag{23}$$

The UAAD across the conterminous United States for $\widehat{Q}_{M3}$ estimators of $Q_{7,10}$ and $Q_{7,2}$ is shown in Fig. 7. It can be seen that the performance of $\widehat{Q}_{M3}$ for estimating both $Q_{7,10}$ and

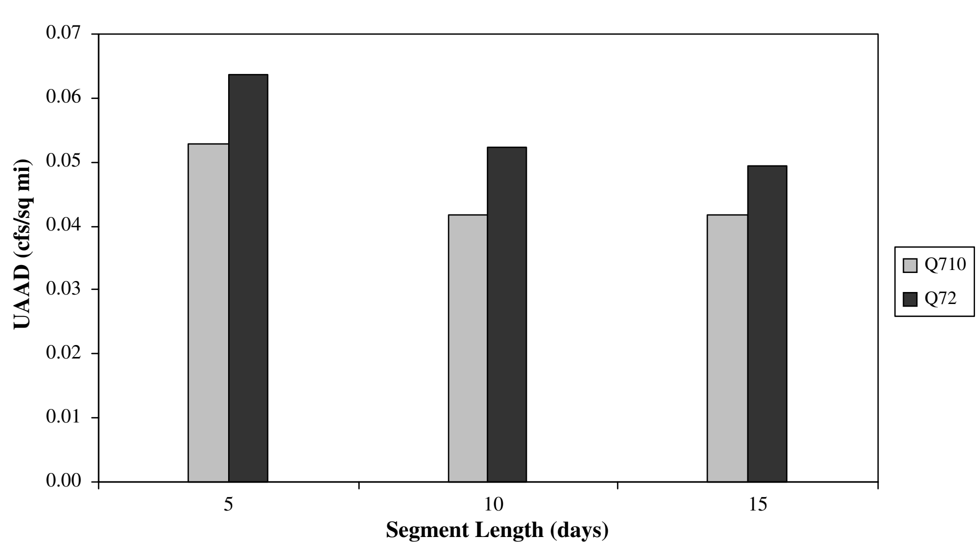

Figure 7 The average Unit Area Absolute Difference (UAAD) for $\widehat{Q}_{M3}$ (the log-space variance-weighted estimator) to estimate the $Q_{7,10}$ (7-day 10-year low flow) and $Q_{7,2}$ (7-day 2-year low flow).

$Q_{7,2}$ increases as the segment length increases, though the marginal gain decreases. From Fig. 7, it appears that the $\widehat{Q}_{M3}$ performs better estimating $Q_{7,10}$ than $Q_{7,2}$ regardless of the segment length. This may be due to the fact that the UAAD measures an absolute difference and the magnitude of $Q_{7,2}$ is larger than $Q_{7,10}$.

Conclusions

The goal of this study was to examine if the performance of the baseflow correlation method can be improved by using multiple nearby-gauged sites instead of a single nearby-gauged site. Seven different estimators of $Q_{7,10}$ and $Q_{7,2}$ using multiple gauged sites were evaluated and compared with the estimator using a single nearby-gauged site. A delete-one cross-validation resampling experiment with more than 1300 USGS Hydro-Climatic Data Network (HCDN) stream sites was conducted to evaluate the proposed estimators. The results indicate that when 10 or less baseflow measurements are available at the partial record site, the multiple site estimators outperform the single site estimator, though for more than 10 baseflow measurements, the single site estimator performs as well as the multiple site estimators. The variance-weighted average in log-space of estimators using single gauged site information ($\widehat{Q}_{M3}$) appears to perform best among the multiple site estimators, though the performance difference among the multiple site estimators was not significant. An upper bound approximation of the variance of $\widehat{Q}_{M3}$ was also derived. As expected, the performance of the baseflow correlation method with multiple gauged sites is improved as the number of baseflow measurements increases. Our final recommendation is to employ the multiple site estimator $\widehat{Q}_{M3}$ when 10 or fewer baseflow measurements are available at the partial record site; for more than 10 baseflow measurements to employ the single site estimator. Ongoing research is examining the performance of these estimators compared to regional regression approaches in smaller study areas with a greater density of gauge sites.

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