Probability Distribution of Low Streamflow Series in the United States

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Abstract: Estimates of low streamflow statistics are required for a variety of water resource applications. At gauged river sites, the estimation of low streamflow statistics requires estimation of annual n-day minimum streamflows, selection of a probability distribution to describe annual minimums, and estimation of the distribution's parameters. Using L-moment diagrams, the ability of various probability distributions to describe low streamflow series was examined at 1,505 gauged river sites in the United States. A weighted distance statistic was developed to compare the goodness-of-fit of different probability distributions for describing low streamflow series. Compared to perennial streamflow sites, a shift of L-moments ratios was observed at intermittent river sites where discharge is sometimes reported as zero. An analytical experiment compared the observed shifts in L-moment ratios at intermittent sites with theoretical L-moment ratio shifts for a number of real- and log-spaced probability distributions. Results of these experiments indicate that Pearson Type III and the 3-parameter lognormal distributions should be the recommended distributions for describing low streamflow statistics in the United States at intermittent and nonintermittent (perennial) sites, respectively.

Introduction

Low streamflow estimates are crucial for: (1) water quality management; (2) issuing or renewing National Pollution Discharge Elimination System (NPDES) permits; (3) planning water supplies, hydropower, cooling and irrigation systems; and (4) assessing the impact of prolonged droughts on aquatic ecosystems. It has been over 20 years since the ASCE Hydraulics Divisions Task Committee on Low-Flow Evaluation, Methods, and Needs (Task Committee 1980) recommended "establish[ing] standard procedures for low-flow measurements and analysis." Unfortunately, such standard procedures have not been developed, and much ambiguity still exists regarding the recommended procedures for estimating low streamflow statistics at both gauged and ungauged stream sites. This study addresses the question: What probability distribution should one employ for a frequency analysis of low streamflows in the United States?

The most widely employed low streamflow statistic in the United States is the 7-day, 10-year, low-flow, $Q_{7,10}$ (Riggs 1980). When a sufficiently long discharge record is available at a river site, low-flow statistics, such as the $Q_{7,10}$, can be obtained by a frequency analysis (Riggs 1968a,b, 1972). The true distribution of minimum river flows is not known, yet one must assume the frequency of low streamflow series can be adequately modeled using a particular probability distribution.

The few studies that have investigated fitting probability distributions to low streamflow series have not arrived at a consensus. Tasker (1987) used a bootstrap resampling experiment of low streamflow quantile estimators to compare the relative performance of four different probability distributions: (1) 3-parameter Weibull (W3); (2) log-Pearson Type III (LP3); (3) log Boughton; and (4) Box-Cox. Analyzing 20 rivers in Virginia, Tasker (1987) recommended the W3 or LP3 for describing the frequency of 7-day annual minimum streamflow series. Condie and Nix (1975) based their "goodness-of-fit" test on the ability of a distribution to produce an acceptable lower bound within the range from zero to the minimum observed flow. Condie and Nix (1975) concluded that the W3 is an appropriate probability distribution for fitting low flows at Canadian rivers. Vogel and Kroll (1989) used a regional probability plot correlation coefficient (PPCC) test to examine the performance of various probability distributions in describing 7-day annual minimum streamflow series in Massachusetts. Based on an analysis of 23 sites, they recommended the 2- and 3-parameter lognormal (LN2 and LN3), LP3, or W3 distributions. Önöz and Bayazit (1999) used a PPCC test to examine the fit of various probability distributions to low flows of varying durations at 16 European rivers, and they recommended the Generalized Extreme Value (GEV) distribution. In two older studies, Matalas (1963) recommended the W3 and the LP3 for fitting 1-day and 7-day low streamflows by analyzing 34 rivers across the United States, while Joseph (1970) recommended the gamma distribution (GAM) for 14-day low flows in Missouri. Vogel and Kroll (1989) and Delleur et al. (1988) provide comprehensive reviews of studies that compared various probability distributions and parameter estimator procedures for fitting low streamflow series.

In general, the United States Geological Survey (USGS) uses a LP3 distribution to describe annual minimum streamflow series, as evidenced by its use in a variety of USGS studies (Barnes

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1986; Tasker 1989; Wandle and Randall 1993; Rumenik and Grubbs 1996). Durrans and Tomic (1996) also advocate the use of the LP3 distribution for frequency analysis of lowflows in the state of Alabama. The recently developed National Drought Atlas (Werick 2000) uses a Generalized Wakeby distribution to describe annual minimum streamflow series. The Generalized Wakeby distribution, a 4-parameter distribution, is one of the most flexible distributions available, because it can mimic the shape of most 3-parameter probability distribution functions. Another approach is to derive the probability distribution of annual minimum low flow series from a physically based hydrologic model (Gottschalk et al. 1997).

One problem with a number of earlier studies was the small number of sites examined. Pearson (1995) analyzed 1-day annual minimum streamflows at over 500 river sites in New Zealand, concluding that no single 2- or 3-parameter distribution provided an adequate fit. Vogel and Wilson (1996) performed a similar study to that of Pearson, examining the performance of various probability distributions in describing 1-day annual, minimum, low flows at 1,455 river sites across the United States and recommended the P3 distribution. Unlike all the other studies mentioned, both Pearson (1995) and Vogel and Wilson (1996) used L-moment diagrams to compare the goodness-of-fit of probability distributions in a region. Clausen and Pearson (1995) used L-moment statistics at 44 sites in New Zealand to show that low flow duration and magnitudes follow a LN3 distribution.

L-moment ratio estimators (L-cv, L-skew and L-kurtosis) were introduced by Hosking (1990) and are derived from probability weighted moment estimators (Landwehr et al. 1979). L-moment ratio estimators are linear estimators and do not suffer from the excessive bias associated with product-moment ratio estimators (Vogel and Fennessey 1993). The benefits of L-moment ratio estimators (such as near normality and unboundedness) have been discussed by many authors (Hosking 1990, 1992; Hosking and Wallis 1997; Vogel and Fennessy 1993). Hosking and Wallis (1997) provide a thorough description of L-moment estimators.

L-moments have been used in a wide variety of goodness-of-fit analyses. For instance, Fill and Stedinger (1995) showed that the power of an L-moment κ test for the Gumbel distribution is greater than that of a PPCC test. Because of the unboundedness and near normality of L-moment estimators, L-moment goodness-of-fit tests have been developed for a number of different distributions, such as the GEV (Chowdhury et al. 1991) and normal (Hosking 1990). Using 42, long-term, gaging stations, Chow and Watt (1994) showed the difficulty of using L-moment diagrams as a goodness-of-fit measure when the number of sites is small. Recently, Caruso (2000) used both a Kolmogorov-Smirnov test and an L-moment ratio test to compare the fit of various probability distributions to low flow series at 21 rivers in New Zealand. Caruso's L-moment test was based on the closeness of the record length-weighted mean values of the sample L-moment ratios to the L-moment ratios of the underlying distribution.

Our investigation employs the USGS's Hydro-Climatic Data Network (HCDN) (Slack and Landwehr 1993), the same data set used by Vogel and Wilson (1996). The HCDN database contains historic daily average streamflows at over 1,500 USGS gauged river sites spatially distributed throughout the United States. In the United States, this database contains streamflow of the highest available quality (Slack and Landwehr 1993).

The goal of this study is to determine appropriate probability distributions for describing annual, minimum, low streamflow series throughout the United States. Unlike Vogel and Wilson (1996), who examined 1-day annual minimum streamflows, we focus our attention on the 7-day, annual minimum streamflows. The 7-day annual minimums are the streamflow series most often analyzed in practice, such as in developing a $Q_{7,10}$ estimate. Summary results are also presented for 1-day and 30-day annual minimum series. A weighted distance measure is developed to reduce any subjective bias from visually interpreting L-moment diagrams. The weighted distance measure is examined both regionally and nationally, and the impact of intermittent river sites on the L-moment analysis is examined. We also employ recently introduced L-moment relations for censored data (Zafirakou-Koulouris et al. 1998) to examine the behavior of low flow series at intermittent river sites.

Development of L-Moment Diagrams

An L-moment diagram provides a visual comparison of sample estimates to population values of the L-moment ratios L-cv, L-skew, and L-kurtosis (Stedinger et al. 1993). A distinct relationship between L-moment ratios exists for various probability distributions. For instance, if one was to examine a 2-parameter distribution, a unique relationship exists between L-cv and L-skew. If a data series is described by a specific 2-parameter distribution, one expects sample L-cv and L-skew estimates to cluster around the L-cv/L-skew relationship for that specific 2-parameter distribution. Vogel and Fennessey (1993) show that L-moment ratio diagrams are always preferred to product moment ratio diagrams for analyzing the goodness-of-fit of a probability distribution to observations.

Data Analyzed

Here, L-moment diagrams are developed using unbiased L-moment estimators (Hosking 1990). Unbiased L-moment estimators are recommended in practice (Stedinger et al. 1993; Vogel and Fennessey 1993; Hosking and Wallis 1995). As mentioned previously, the USGS's HCDN database was used in this experiment. Only the 1,560 sites with flows acceptable on a daily time-step (as defined within the HCDN database) were employed. In addition, only sites with at least 4 non-zero d-day annual minimum flows were included in the analysis. For 7-day annual minimum flows, this criterion removed 55 sites from the analysis. Unlike Vogel and Wilson (1996), no discordancy measures (Hosking and Wallis 1993) were used prior to the L-moment analysis to eliminate sites with unusual sample statistics. Many of the sites Vogel and Wilson removed due to discordancy were intermittent sites; in this experiment such sites proved important when interpreting shifts in L-moment diagram trends, as will be discussed later. Thus for 7-day annual minimum flows, 1,505 of the HCDN sites were analyzed.

L-Moment Diagram Comparisons

Fig. 1 illustrates sample estimates of L-cv versus L-skew for 7-day annual minimum low streamflows in the conterminous United States. Also plotted on this figure are the L-cv/L-skew relationships for three 2-parameter probability distributions: log-normal (LN2); gamma (GAM); and Weibull (W2). The plotted relationships for the 2-parameter distributions were based on the polynomial approximations developed by Vogel and Wilson (1996).
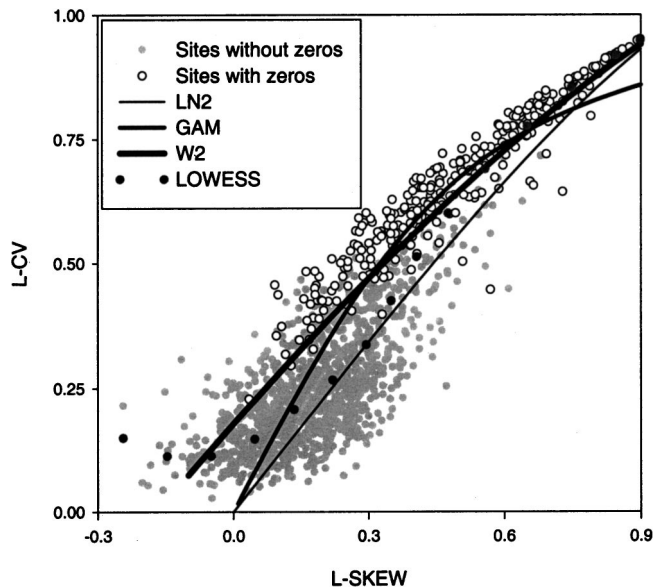
Fig. 1. L-cv versus L-skew for conterminous USA

Fig. 3. USGS water resource regions

Fig. 2 illustrates sample L-kurtosis versus sample L-skew for 7-day annual minimum streamflow across the conterminous United States. This diagram displays the L-kurtosis/L-skew relationships for four 3-parameter distributions: generalized extreme value (GEV), Weibull (W3), lognormal (LN3), and Pearson Type III (P3). These curves were drawn based on the polynomial approximations given by Hosking (1991) and Stedinger et al. (1993). It is important to note the log-Pearson III (LP3) distribution, whose log-spaced L-moment ratios follow the same trends as those of a P3 distribution, is not analyzed in this experiment. This is because a unique relationship does not exist between the real-space L-moment ratios for the LP3 distribution.

It is difficult to ascertain from Figs. 1 and 2 what is the most appropriate distribution to model 7-day annual minimum low streamflows across the conterminous United States. As a goodness-of-fit criterion, Vogel and Wilson (1996) fit a locally
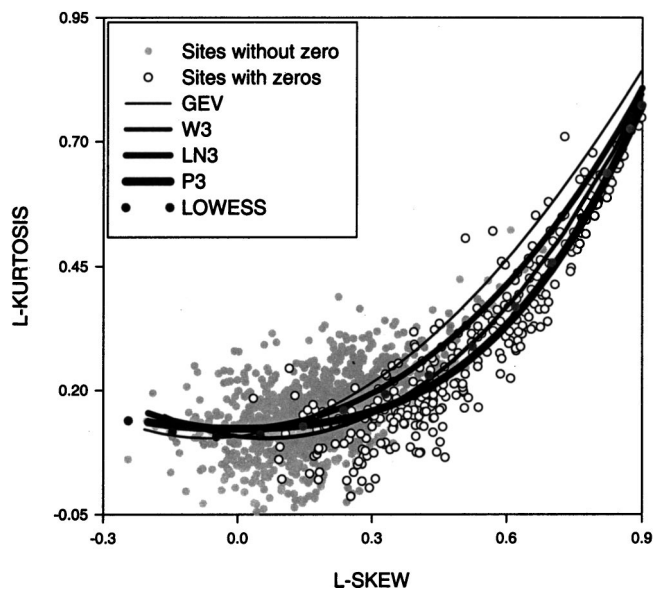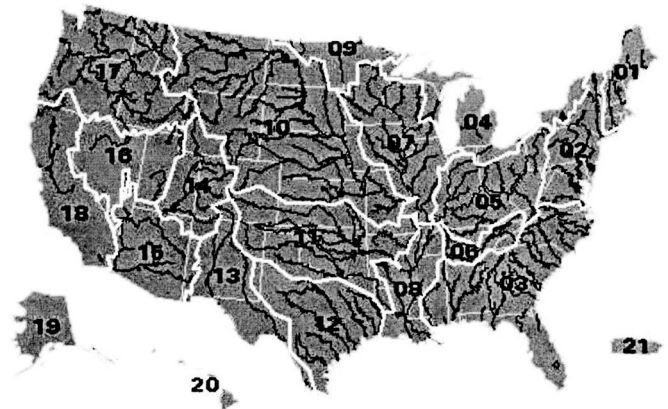
weighted scatter-plot smoothing (LOWESS) curve (Cleveland 1979) to the sample L-moment ratios, and then compared the LOWESS curve to the L-moment relationships for the different distributions mentioned above. The LOWESS curve, though, is somewhat subjective in nature. The amount of smoothing captured by the LOWESS curve is controlled by altering the LOWESS parameters. In addition, if one was instead to fit the LOWESS curve by interchanging the x-axis and y-axis (L-skew is generally plotted on the x-axis in L-moment diagrams), a different curve would be obtained. The LOWESS curve with the suggested parameterization (tricube weighting function, polynomial order of 1, and a smoothing parameter of 0.5) is also included in Figs. 1 and 2.

To avoid difficulties with the visual interpretation of the L-moment diagrams, a performance measure was developed. This measure, the average weighted orthogonal distance (AWOD), is defined as

$$\text{AWOD} = \frac{\sum_{i=1}^{N} \text{RL}_i d_i}{\sum_{i=1}^{N} \text{RL}_i} \qquad (1)$$

where RL_i=record length at site i; d_i=orthogonal distance between the sample L-moments at site i and the L-moment relationship for a specific distribution; and N=number of sites. The AWOD measures the average weighted distance between sample L-moment ratios and a probability distribution's theoretical L-moment ratio relationship. The closer the sample L-moment ratios are to a probability distribution's L-moment ratio relationship the smaller the AWOD, which would indicate a better choice of distribution for describing the low flow series. The AWOD is similar to the measure used in Caruso's (2000) L-moment diagram test. To compare the relative performance of each method relative to the best method, a performance ratio (PR) was determined as

$$\text{PR}_{\text{Method}} = \text{AWOD}_{\text{Best Method}} / \text{AWOD}_{\text{Method}} \qquad (2)$$

The best method will have the smallest AWOD and, thus, will have a PR of 1. All other methods will have a PR between 1 and 0.

The PR was estimated for 7-day annual minimum streamflows in each of the USGS's 21 water resource regions. These water resource regions are shown in Fig. 3. Only sites with no zero observations were included in the calculation of performance ratios, because Figs. 1 and 2 document that sample L-moment ratios
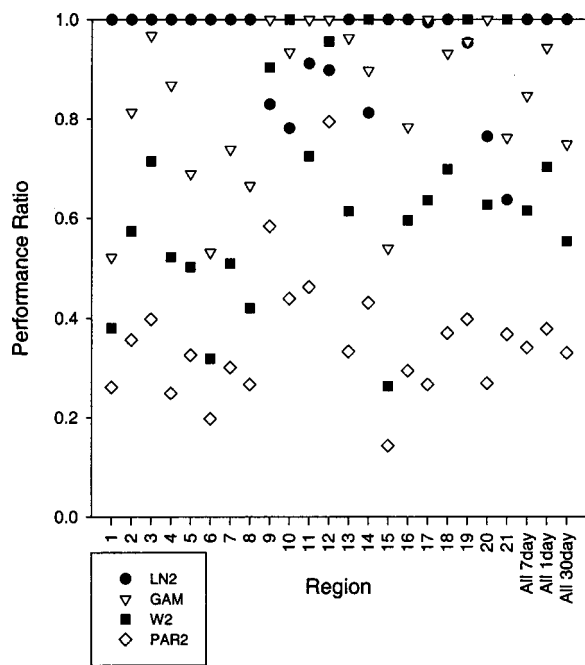

Fig. 2. L-kurtosis versus L-skew for conterminous USA

Fig. 4. Performance ratios for 2-parameters distributions: No zeros
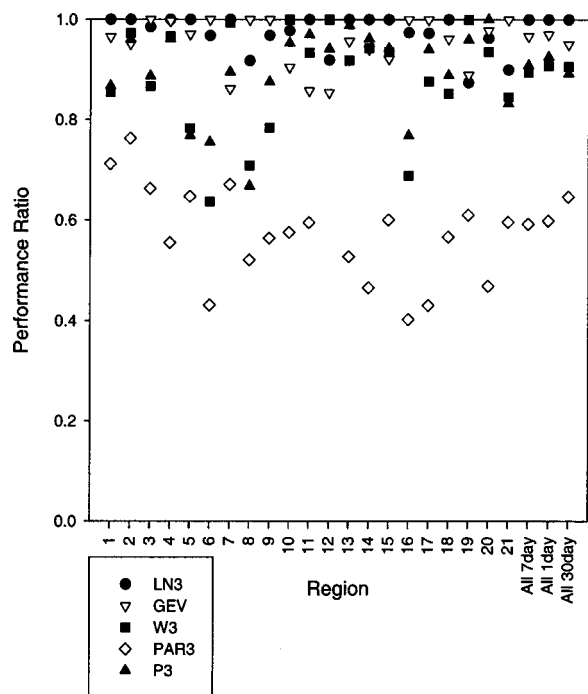

Fig. 5. Performance ratios for 3-parameter distributions: No zeros

at sites with and without zeros behave quite differently. Furthermore, the theoretical L-moment curves in Figs. 1 and 2 are only consistent with the L-moment estimates for the sites with non-zero observations. Fig. 4 contains the performance ratio for four, 2-parameter distributions—LN2, GAM, W2, and Generalized Pareto (PAR2). Results are presented for each water resource region, as well as for all sites within the conterminous United States combined. Thus, "All 7 day" in Fig. 4 refers to 7-day annual minimums at all sites within water resource regions 1 through 18. In comparing 2-parameter distributions, it appears that the LN2 distribution performs slightly better than the GAM distribution, providing the smallest AWOD (PR=1) in 12 of the 21 regions and with the smallest AWOD across all regions. The GAM was preferred in only 5 of the regions, while the W2 was preferred in only 3 of the regions. The generalized pareto (PAR2) distribution always performed poorly compared to the other distributions analyzed. Overall, the LN2 distribution performed best, followed by the GAM, W2 and PAR2 in that order. Also included in Fig. 4 are the 2-parameter distribution results for the 1-day and 30-day annual minimum low streamflows across the conterminous United States. For 1-day, 7-day, and 30-day annual minimums, the LN2 distribution had the smallest AWOD across all regions.

Fig. 5 contains the PR for 7-day annual minimum streamflows using five 3-parameter distributions—LN3, GEV, W3, Pareto (PAR3), and P3. Results are presented for each water resource region, as well as for all sites in the conterminous United States. In comparing 3-parameter distributions, it appears that the GEV and LN3 distributions perform equally as well, with the W3 and P3 providing a slightly poorer fit and PAR3 performing very poorly. The LN3 was preferred in 10 regions and was the preferred distribution across the conterminous United States, while the GEV was preferred in 8 regions, the W3 in 3 regions, and the P3 in 1.

Also included in Fig. 5 are the 3-parameter distribution results for the 1-day and 30-day annual minimum low streamflows across the conterminous United States. For 1-day, 7-day, and 30-day

annual minimums, the LN3 distribution had the smallest AWOD across all regions, followed closely by the GEV distribution.

Similar to the results of Pearson (1995), no one distribution provides a superior fit to annual minimum streamflow series across all regions of the United States. In general, 2-parameter distributions do not provide enough flexibility to describe a wide range of distributional shapes, and probably should be avoided in low streamflow frequency analyses. Results from this first analysis indicate that of the 3-parameter distributions analyzed, the LN3 distribution appears to provide the best fit to low streamflow series at nonintermittent river sites throughout the United States, followed by the GEV, P3 and W3, in that order. Of the 2-parameter distributions examined, the LN2 distribution performed best for the sites with no zero streamflow observations.

Impact of Intermittent Gauging Sites

In Figs. 1 and 2, a distinction was made between sites with and without 7-day annual minimums calculated as zero, with nonzero sites represented using solid circles and sites with zeros represented using unfilled circles. In both of these figures, the sample L-moment ratios for the intermittent sites appear to follow a different trend, with a general shift to the upper right, which indicates an increase in all three of the sample L-moment ratios. Figs. 6, 7, and 8 contain plots of the three sample L-moment ratios versus the percentage of zeros across all 21 regions. These plots all have a trend with a positive slope, which indicates an increase in L-moment ratios with an increase in the percentage zeros.

While one may consider the occurrence of zero discharges to be remote, of the 1,505 HCDN sites analyzed above, 304 (20%) contained 7-day annual minimum streamflows calculated as zero. Typically, all discharge values below a measurement threshold are reported as zero. The measurement threshold is often a function
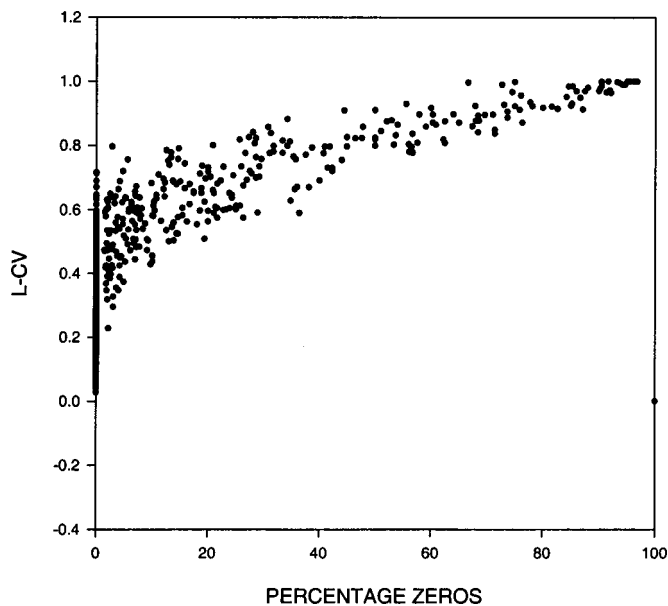
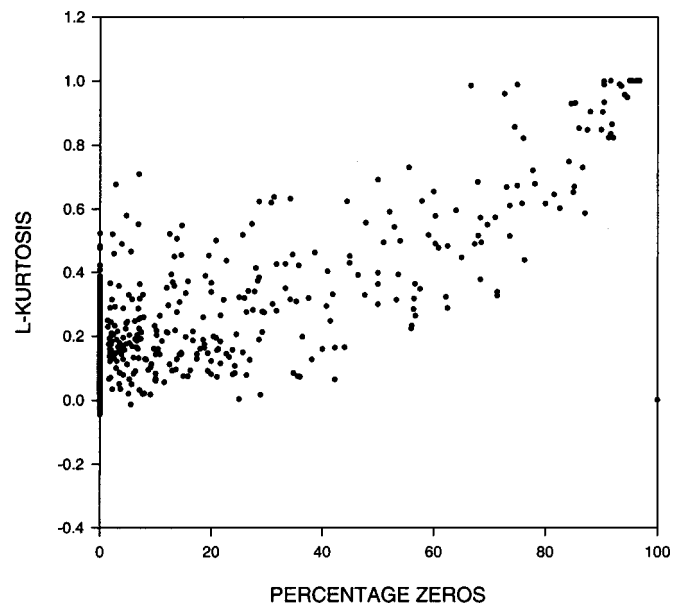**Fig. 6.** L-cv versus percentage of zeros for conterminous USA

Fig. 8. L-kurtosis versus percentage of zeros for conterminous USA

of the physical characteristics of the stream at the gauge location, and generally ranges from 0.01 to 0.1 cfs. Zero discharges at river sites are often referred to as censored data (Kroll and Stedinger 1996; Durrans et al. 1999), and the percentage of zeros is referred to as the censoring percentage. Estimators for censored distributions have been investigated for both log-spaced (Kroll and Stedinger 1996) and real-spaced (Wang, 1990a, b, 1996) distributions.

There are many potential models of intermittent streamflows, and various estimators of streamflow statistics take advantage of these models. One procedure used for estimating low streamflow quantiles at intermittent river sites is to employ a conditional probability adjustment (CPA) (Tasker, 1989), a methodology originally developed for floods (Jennings and Benson 1969; Haan 1977). The CPA models the low-flow process as a mixed distri-

bution with a point mass at zero and a continuous distribution for the non-zero observations. In Figs. 1 and 2, all flows (including those recorded as zero) were used to develop sample L-moment ratio estimators. Figs. 1 and 2 were reproduced using only the non-zero values at intermittent sites to develop sample L-moment ratios. If the continuous distribution that describes the non-zero observations at intermittent sites follows the same L-moment ratio pattern as observed at sites with no zeros, we should see no difference in L-moment patterns for the intermittent sites and nonintermittent (no zeros) sites. Contrary to this, a shift similar to that seen in Fig. 2 was also observed (this figure is not included here). This indicates non-zero discharges at intermittent sites do not follow a similar L-moment ratio pattern as those from nonintermittent sites, which may indicate that either a censored or truncated distribution should be used to describe the non-zero observations at intermittent sites.

The next section presents an analysis of L-moment diagram shifts due to censoring. Of interest is whether the observed shifts in L-moment diagrams, when censoring occurred, can be reproduced by analyzing the theoretical shifts due to censoring in L-moments for various probability distributions.
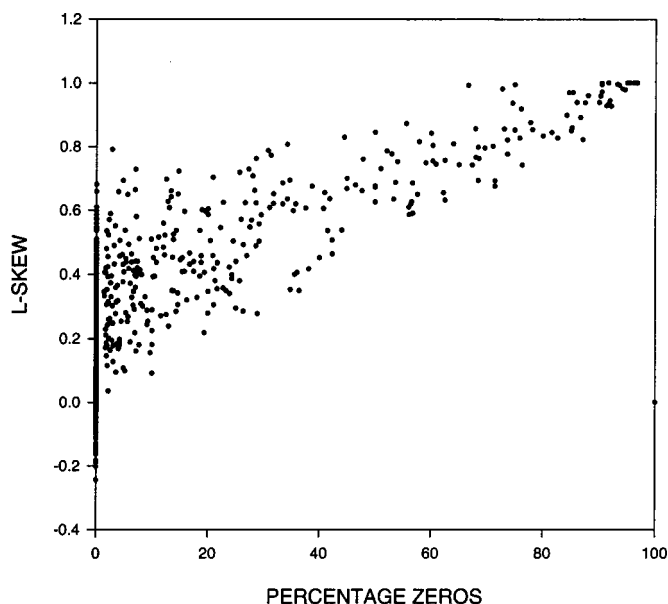
L-Moment Ratio Shifts due to Censoring

Hosking (1995) introduced two types of probability weighted moment (PWM) estimators for right censored samples, type A and type B. Right-hand censoring is common in manufacturing applications, which formed the impetus for Hosking's (1995) work. Zafirakou-Koulouris et al. (1998) introduced PWM estimators for left censored observations, which differ from the expressions for right censored observations introduced by Hosking (1995). Type A censored PWMs are equivalent to the L-moments of the uncensored observations. Type B occur when the censored observations are replaced by some nominal value (typically the censoring threshold), and the PWMs are calculated from the "completed sample". In this analysis, we have calculated sample L-moments treating the zero observations as zeros, which is equivalent to type B censored PWMs where the censored observations are replaced by zero.
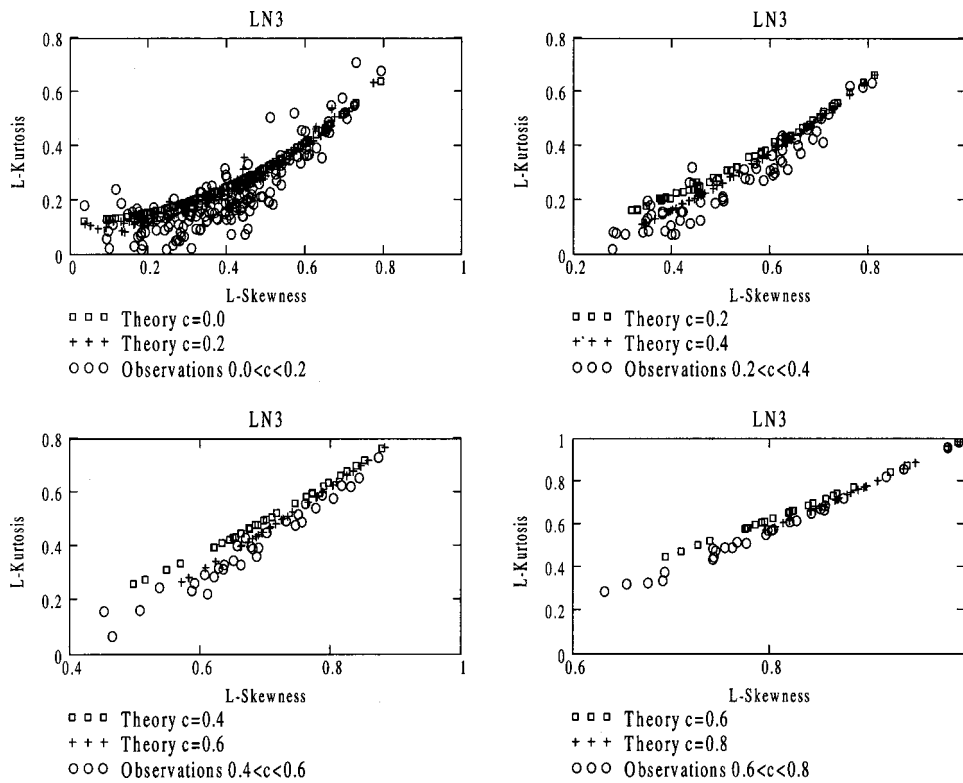

Fig. 7. L-skew versus percentage of zeros for conterminous USA

Fig. 9. Theoretical shifts in L-moment ratios for a censored 3-parameter lognormal distribution

Zafirakou-Koulouris et al. (1998) investigated shifts in L-moment diagrams of left censored data sets for numerous probability distributions. They introduced theoretical expressions for type A and type B left censored PWMs, which relate the censoring percentage, c, to the censored PWMs. Using this information, one can theoretically derive the impact of censoring on L-moments for various censored probability distributions. Using the notation of Zafirakou-Koulouris et al. (1998) for left censoring, the inverse cumulative distribution function or quantile function for type B censored PWMs can be expressed as

$$y^{B'}(u) = \begin{cases} x(c) & 0 < u < c \\ x(u) & c \leq u < 1 \end{cases} \tag{3}$$

The quantile function $y^{B'}$ expresses the magnitude of an event (here a 7-day annual minimum lowflow) as a function of its non-exceedance probability (Hosking and Wallis 1997). The $x(c)$ in Eq. (3) represents the value assigned to the censored observations. Zafirakou-Koulouris et al. (1998) showed that left censored type B PWMs can be defined as

$$\beta_r^{B'} = x(c) \frac{c^{r+1}}{r+1} + \int_c^1 u^r x(u) du \tag{4}$$

When the censored observations $x(c)$ are replaced by zero, the first term of (4) drops out leaving

$$\beta_r^{B'} = \int_c^1 u^r x(u) du \tag{5}$$

The quantile function for a specific distribution allows examination of how PWMs, L-moments, and L-moment ratios change theoretically under various censoring scenarios. Hosking and Wallis (1997) provide quantile functions for many commonly employed probability distributions.

Four probability distributions were considered—LN3, GEV, P3, and the 4-parameter Generalized Wakeby distribution (W4). In the following experiment, only the 304 sites with censored observations were employed, because those sites are hypothesized to follow a different probabilistic trend than sites with no zeros. Using the sample L-moments for the sites with zeros, the parameters for the four probability distributions were estimated, resulting in 304 different parameter sets for each distribution. The censored sites were grouped based on ranges of censoring: $0 < c < 0.2$; $0.2 < c < 0.4$; $0.4 < c < 0.6$; and $0.6 < c < 0.8$. Within a specific range, once the parameters for a distribution were estimated, the L-moments ratios were then estimated using Eq. (5) at the upper and lower bounds of the censoring range. For instance, within the range of censoring $0.2 < c < 0.4$, all sites with between 20 and 40% of 7-day annual minimums reported as zero were considered. For each distribution, the parameters were then estimated at each of these sites, and then the L-moment ratios were calculated for censoring of 20 and 40%, the bounds of the range.

Figs. 9 through 12 contain plots of L-kurtosis versus L-skew for the four distributions examined. Each figure contains four plots, one for each of the censoring ranges. Each plot contains three different symbols representing both sample and theoretical L-moment ratios. The circles represent the sample L-moments for the sites with censoring within the specific censoring range. These are the same sample L-moments plotted in Fig. 2. The squares represent the theoretical L-moment ratios calculated at the lower end of the censoring range, and the crosses represent theoretical L-moment ratios at the upper end of the censoring range.

Since L-moment ratio estimators are unbiased estimators, we would expect the theoretical L-moment ratios to either encompass the sample L-moment ratios or for there to be a uniform scatter of the sample L-moment ratios around the theoretical L-moment ratios. Figs. 9 and 10 are for the LN3 and GEV distributions. For
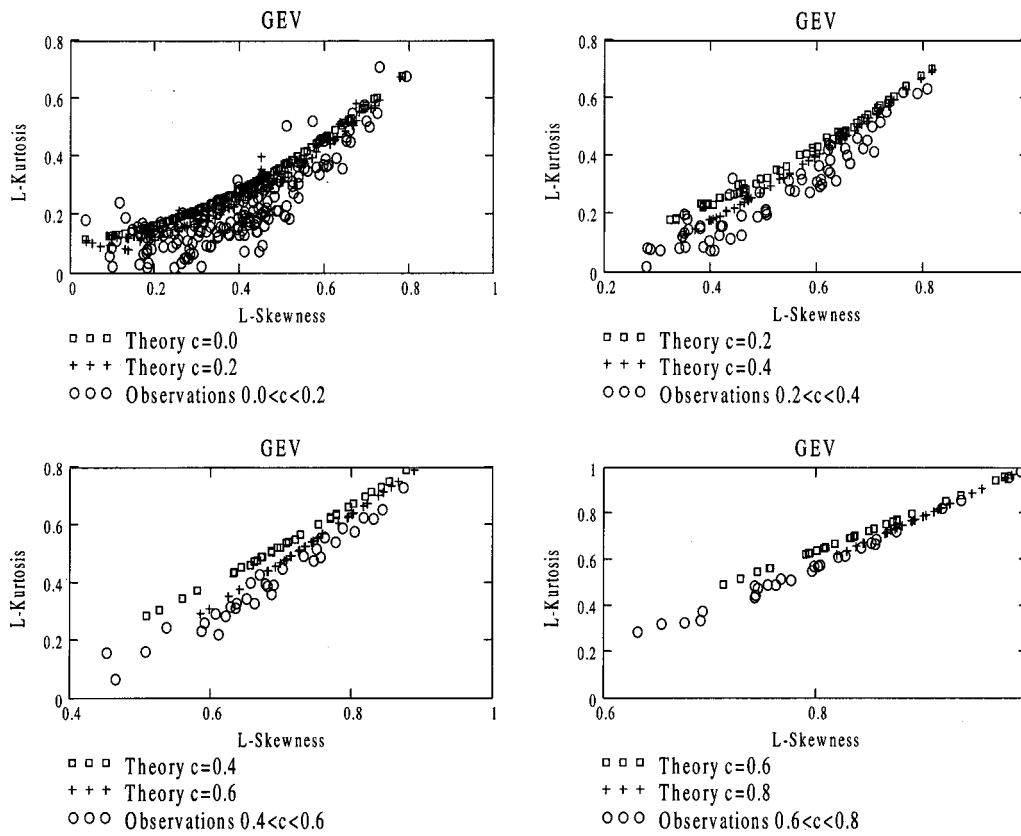
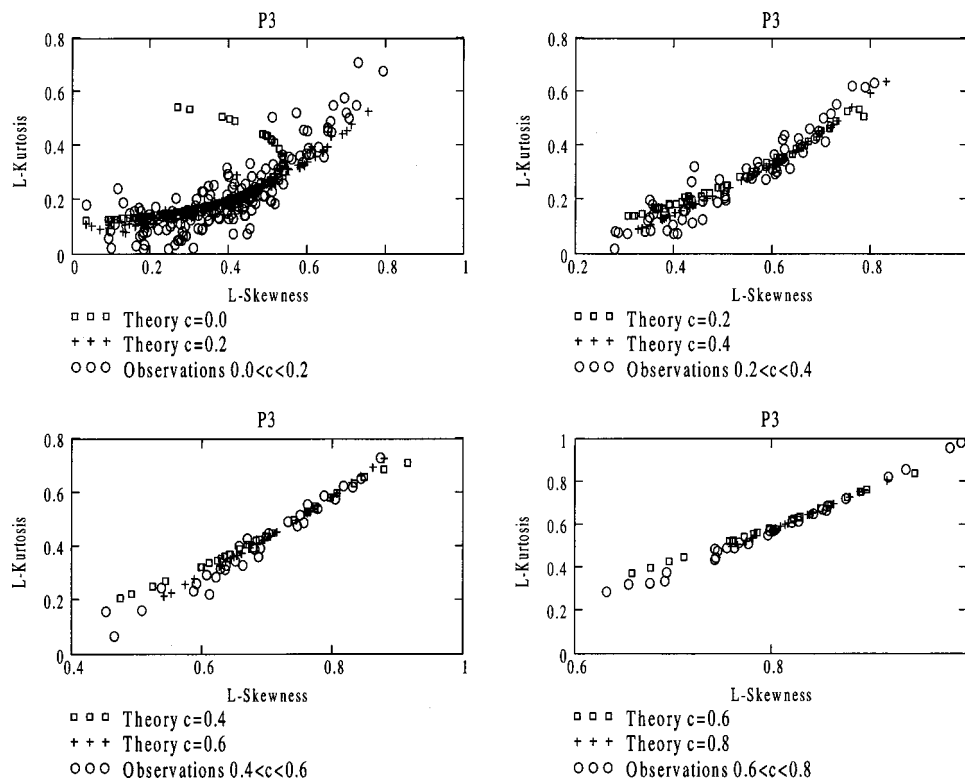**Fig. 10.** Theoretical shifts in L-moment ratios for a censored generalized extreme value distribution

Fig. 11. Theoretical shifts in L-moment ratios for a censored Pearson type III distribution
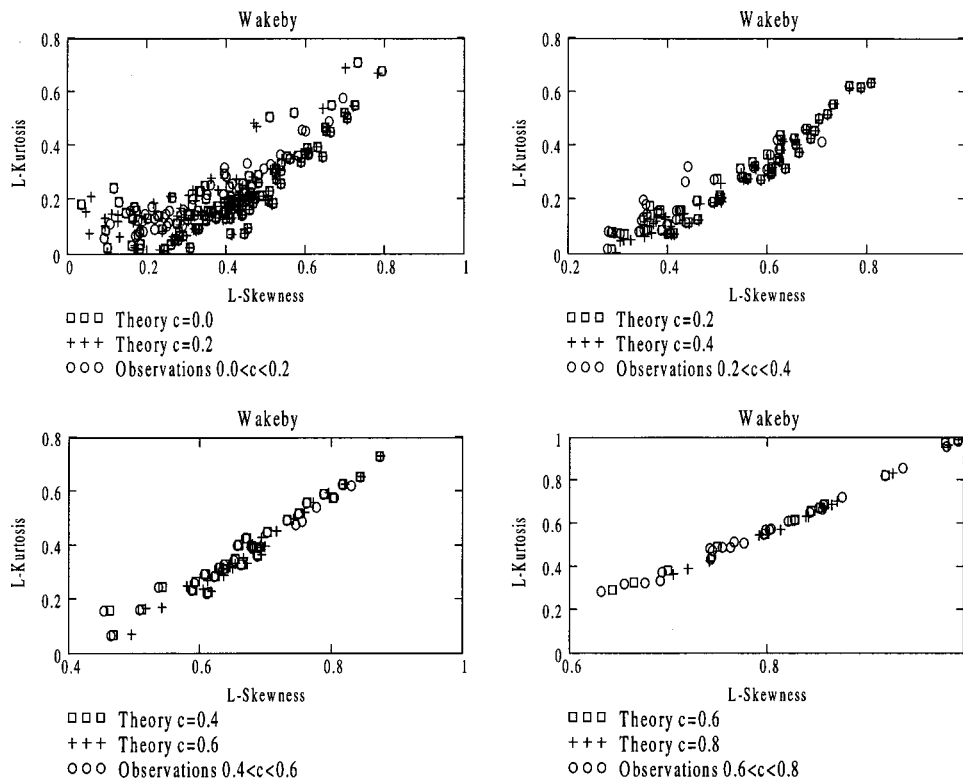
Fig. 12. Theoretical shifts in L-moment ratios for a censored 4-parameter generalized Wakeby distribution

lower censoring ranges, the theoretical curves for these distributions appear to intersect the sample data, yet for censoring above 0.2, there appears to be a strong upward bias of the theoretical curves.

Fig. 11 contains plots for the P3 distribution. In this figure, the upward bias observed for the LN3 and GEV distributions is not apparent at any censoring range. The theoretical curves appear to intersect the sample data as they would be expected to if the data were correctly described by this distribution. This result indicates the trends observed in sample L-moment ratios at intermittent sites are better described by the P3 distribution than by either the LN3 or GEV distributions.

Fig. 12 contains theoretical curves for the W4 distribution, a Generalized Wakeby distribution (lower bound set to zero), at the various censoring ranges. The trends in sample L-moment ratios appear to be well described by the W4 distribution. This result is to be expected, because the Wakeby distribution is an extremely flexible probability distribution that can mimic most of the other probability distribution functions considered.

Conclusions

In this study, L-moment diagrams were constructed with sample L-moment ratios for 1-, 7-, and 30-day annual minimum streamflow series at all HCDN river sites. Although L-moment diagrams are recommended in practice, using visual observation of L-moment diagrams to distinguish between competing distributions proved extremely difficult, even though the sample size was relatively large (approximately 1,500 sites).

L-moment diagrams illustrated that sample L-moment ratios at intermittent and nonintermittent (all nonzero streamflows) sites follow different trends, hence we elected to evaluate these two types of sites separately. For the nonintermittent sites we used a weighted distance performance measure to compare the performance of four 2-parameter and five 3-parameter probability distributions. This performance measure was analyzed both regionally and nationally for the nonintermittent sites. Results indicated that no one distribution provides a superior fit to annual minimum streamflow series at all locations within the United States. Of the 3-parameter distributions analyzed, the 3-parameter lognormal (LN3) distribution provided the best overall fit when intermittent streamflow sites were excluded from the analysis. The GEV distribution performed nearly as well as the LN3 distribution followed by the LP3 and W3 distributions.

Of the HCDN sites analyzed, approximately 20% contained 7-day annual minimum streamflows calculated as zero. At these intermittent river sites, sample L-moment ratios were generally larger than the sample L-moment ratios at nonintermittent sites, with sample L-moment ratios increasing as the percentage of zeros increased. Streamflow discharge below a measurement threshold is typically reported as zero. Such a process produces what is generally referred to as censored data. To investigate L-moment ratio shifts due to censoring, an analytical experiment was conducted to analyze the performance of four distributions—LN3, GEV, P3, and the 4-parameter Generalized Wakeby distribution (W4). Using the 304 sites with intermittent streamflows, the underlying distributions were parameterized and then theoretical shifts in L-moment ratios were calculated for various censoring levels. These theoretical points were then plotted with the sample L-moment ratios for the intermittent sites to see if any of the observed trends were reproduced by these distributions. The P3 and W4 produced theoretical points which uniformly intersected the sample L-moment ratios, while the LN3 and GEV produced points that generally overestimated the L-kurtosis and, thus, appeared upwardly biased. This result indicates the P3 dis-

tribution provided a better description of the censoring process than the GEV or LN3 distributions at the intermittent sites.

Based on these results, the following three conclusions were reached:

1. Even when a large data set is present, it is difficult to distinguish between competing probability distributions using L-moment diagrams.
2. If one distribution was to be employed to describe low streamflow series at nonintermittent sites across the United States, the LN3 distribution is preferred.
3. If one distribution was to be employed to describe low streamflow series at intermittent sites across the United States, the P3 distribution is recommended.

Acknowledgments

References

Barnes, C. R. (1986). "Methods of estimating low-flow statistics for ungaged streams in the lower Hudson River Basin, NY." *U.S. Geological Survey Water Resources Investigations Rep. 85-4070*, Washington, D.C.

Caruso, B. S. (2000). "Evaluation of low-flow frequency analysis methods," *J. Hydrol.,* 39(1), 19–47.

Chow, K. C. A., and Watt, W. E. (1994). "Practical use of L-moments." *Stochastic and statistical methods in hydrology and environmental engineering*, Vol. 1, K. W. Hipel, ed., Kluwer Boston, 55–69.

Chowdhury, J. U., Stedinger, J. R., and Lu, L. H. (1991). "Goodness-of-fit tests for regional generalized extreme value flood distributions," *Water Resour. Res.,* 27(7), 1765–1176.

Clausen, B., and Pearson, C. P. (1995). "Regional frequency-analysis of annual maximum streamflow drought," *J. Hydrol.,* 173(1–4), 111–130.

Cleveland, W. S. (1979). "Robust locally weighted regression and smoothing scatterplots," *J. Am. Stat. Assoc.,* 74, 829–836.

Condie, R., and Nix, G. A. (1975). "Modeling of low flow frequency distributions and parameter estimation." *Int. Water Resource Symposium, Water for Arid Lands*, Teheran, Iran.

Delluer, J. W., Rao, A. R., and Bell, J. M. (1988). "Criteria for the determination of minimum streamflows." *Tech. Rep. CE-HSE-88-6*, School of Civil Engineering., Purdue Univ., West Lafayette, Ind.

Durrans, S. R., and Tomic, S. (1996). "Regionalization of low-flow frequency estimates: An Alabama case study," *Water Resour. Bull.,* 32(1), 23–37.

Durrans, S. R., Ouarda, T. B. M. J., Rasmussen, P. F., and Bobée, B. (1999). "Treatment of zeroes in tail modeling of low flows," *J. Hydrologic Eng.,* 4(1), 19–27.

Fill, H., and Stedinger, J. R. (1995). "Homogeneity tests based upon Gumbel distribution and a critical appraisal of Dalrymple's test," *J. Hydrol.,* 166, 81–105.

Gottschalk, L., Tallaksen, L. M., and Perzyna, G. (1997). "Derivation of low flow distribution functions using recession curves," *J. Hydrol.,* 194(1–4), 239–262.

Haan, C. T. (1977). *Statistical methods in hydrology*, Iowa State Univ. Press, Ames, Iowa.

Hosking, J. R. M. (1990). "L-moments: Analysis and estimation of distributions using linear combinations or order statistics," *J. R. Stat. Soc. Ser. B. Methodol.,* 52, 105–124.

Hosking, J. R. M. (1991). "Approximations for use in constructing L-moment ratio diagrams." *Res. Rep. RC 16635 (#73810)*, IBM Research Div., T. J. Watson Research Center, Yorktown Heights, N.Y.

Hosking, J. R. M. (1992). "Moment or L moments? An example comparing two measures of distributional shape," *Am. Stat.,* 46(3), 186–189.

Hosking, J. R. M. (1995). "The use of L-moments in the analysis of censored data." *Recent advances in life-testing and reliability*, N. Balakrishnan, ed., CRC, Boca Raton, Fla., 546–560.

Hosking, J. R. M., and Wallis, J. R. (1993). "Some statistics useful in regional frequency Analysis," *Water Resour. Res.,* 29(3), 271–281.

Hosking, J. R. M., and Wallis, J. R. (1995). "Comparison of unbiased and plotting position estimators of L-moments," *Water Resour. Res.,* 31(8), 2019–2025.

Hosking, J. R. M., and Wallis, J. R. (1997). *Regional frequency analysis an approach based on L-moments*, Cambridge Univ. Press, Cambridge, U.K.

Jennings, M. E., and Benson, M. A. (1969). "Frequency curves for annual flood series with some zero events or incomplete data," *Water Resour. Res.,* 5(1), 276–280.

Joseph, E. S. (1970). "Probability distribution of annual droughts," *J. Irrig. Drain.,* 96(4), 461–474.

Kroll, C. N., and Stedinger, J. R. (1996). "Estimation of distributional moments and quantiles for censored data," *Water Resour. Res.,* 32(4), 1,005–1,012.

Landwehr, J. M., Matalas, N. C., and Wallis, J. R. (1979). "Probability weighted moments compared with some traditional techniques in estimating Gumbel parameters and quantiles," *Water Resour. Res.,* 15(5), 1,055–1,064.

Matalas, N. C. (1963). "Probability distribution of low flows." *Professional Paper 434-A*, U.S. Geological Survey, Washington, D.C.

Önöz, B., and Bayazit, M. (1999). "GEV-PWM model for distribution of minimum flows," *J. Hydrologic Eng.,* 4(3), 289–292.

Pearson, C. P. (1995). "Regional frequency analysis of low flows in New Zealand rivers," *J. Hydrol.,* 30(2), 53–64.

Riggs, H. C. (1968a). "Chapter A2: Frequency curves." *U.S. Geological Survey Techniques of Water Resource Investigations*, Book 4, Washington, D.C.

Riggs, H. C. (1968b). "Chapter B3: Regional analyses of streamflow characteristics." *U.S. Geological Survey Techniques of Water Resource Investigations*, Book 4, Washington, D.C.

Riggs, H. C. (1972). "Chapter B1: Low-flow investigations." *U.S. Geological Survey Techniques of Water Resource Investigations*, Book 4, Washington, D.C.

Riggs, H. C. (1980). "Characteristics of low flows," *J. Hydraul. Eng.,* 106(5), 717–731.

Rumenick, R. P., and Grubbs, J. W. (1996). "Methods for estimating low-flow characteristics for ungaged streams in selected areas, northern Florida." *U.S. Geological Survey Water Resources Investigation Rep. 96–4,124*, Washington, D.C.

Slack, J. R., and Landwehr, J. M. (1993). "Hydro-climatic data network (HCDN): A U.S. Geological Survey streamflow data set for the United States for the study of climate variations, 1874–1988." *U.S. Geological Survey Water Resources Investigation Rep., 93–4,076*, Washington, D.C.

Stedinger, J. R., Vogel, R. M., and Foufoula-Georgiou, E. (1993). "Chapter 18: Frequency analysis of extreme events." *Handbook of hydrology*, D. R. Maidment, ed., McGraw-Hill, New York.

Task Committee on Low-Flow Evaluation, Methods, and Needs of the Committee on Surface-Water Hydrology of the Hydraulics Division. (1980). "Characteristics of low flows," *J. Hydr. Div. Am. Soc. Civil Eng.,* 106(5), 717–731.

Tasker, G. D. (1987). "A comparison of methods for estimating low flow characteristics of streams," *Water Resour. Bull.,* 23(6), 1,077–1,083.

Tasker, G. D. (1989). "Regionalization of low flow characteristics using logistic and GLS regression." *New directions for surface water modeling*, M. L. Kavvas, ed., IAHS Publication No. 181, 323–331.

Vogel, R. M., and Fennessey, N. M. (1993). "L-Moment diagrams should replace product moment diagrams," *Water Resour. Res.,* 29(6), 1,745–1,752.

Vogel, R. M., and Kroll, C. N. (1989). "Low-flow frequency analysis using probability-plot correlation coefficients," *J. Water Resour. Plan. Manage.,* 115(3), 338–357.

Vogel, R. M., and Wilson, I. (1996). "Probability distribution of annual maximum, mean, and minimum streamflows in the United States," *J. Hydrologic Eng.,* 1(2), 69–76.

Wandle, S. W., and Randall, A. D. (1993). "Effects of surficial geology, lakes and swamps, and annual water availability on low flows of streams in central New England, and their use in low-flow estimation." *U.S. Geological Survey Water Resources Investigation Report 93–4092*, Washington, D.C.

Wang, Q. J. (1990a). "Estimation of the GEV distribution from censored samples by method of partial probability weighted moments," *J. Hydrol.,* 120, 103–114.

Wang, Q. J. (1990b). "Unbiased estimation of probability weighted moment and partial probability weighted moments for systematic and historical flood information and their application to estimating the GEV distribution," *J. Hydrol.,* 120, 115–124.

Wang, Q. J. (1996). "Using partial probability weighted moments to fit the extreme value distributions to censored samples," *Water Resour. Res.,* 32(6), 1,767–1,771.

Werick, B. (2000). *National Drought Atlas*, U.S. Army Corps of Engineers, Institute for Water Resources, ⟨http://www.wrsc.usace.army.mil/iwrnew/Atlas/Atlasintro.htm⟩.

Zafirakou_Koulouris, A., Vogel, R. M., Craig, S. M., and Havermeier, J. (1998). "L-moment diagrams for censored observations," *Water Resour. Res.,* 34(5), 1,241–1,249.