Thermodynamics Review

In the study of thermodynamics we focus our attention on either

- A fixed mass of a substance - **closed system**, control mass, or **system** or
- A region in space - **control volume** or open system.

Everything external to the boundaries of the control mass or control volume forms the surroundings.

I**solated system** - no mass or energy crosses the boundaries of the system.

Uniform state - properties are not dependent on the spatial coordinates.

Extensive Property - Value of the property of a substance in a uniform state is dependent on the mass of the substance considered, the property is an **extensive property**.

Intensive Property: Property value is independent of the mass. Volume is an extensive property: temperature and pressure are intensive properties.

Intensive properties derived by dividing the extensive properties by the mass. Examples, specific volume

Solid: Shape is not defined by the container, and volume of a fixed mass is finite.

Liquid: Shape is defined by the container, has the possibility of a free surface and volume of a fixed mass is finite.

Gas: Shape is defined by the container, cannot have a free surface, and occupies all the space available - the entire space of the container.

Example of the three phases of a pure substance: Ice (solid), water (liquid), and steam (gas).

WORK TRANSFER

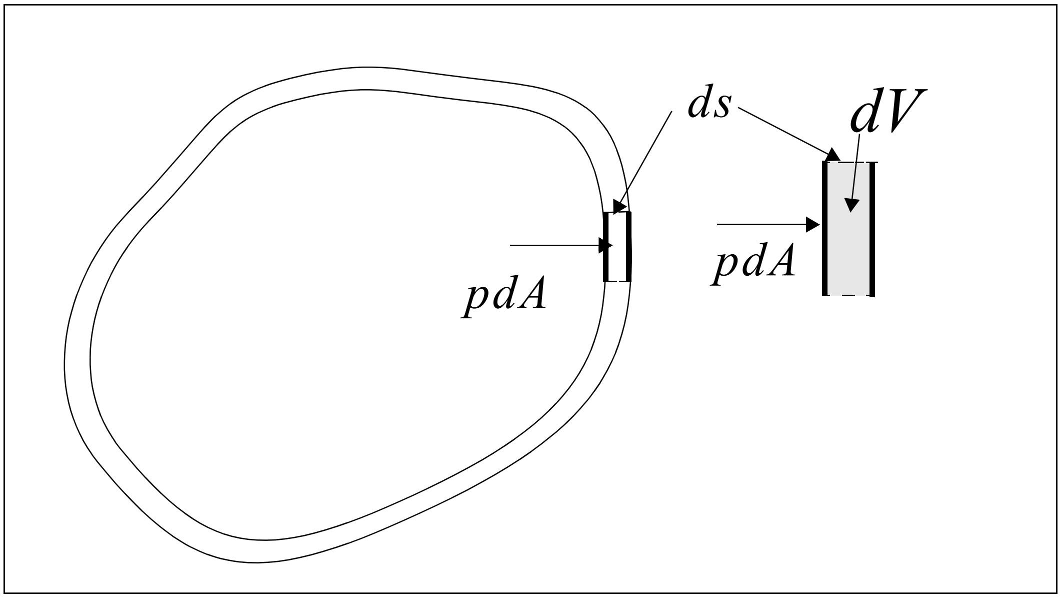
$$\text{Increase in the kinetic energy} \quad = \frac{1}{2}m(V_2^2 - V_1^2)$$

where subscript 2 indicates the final velocity and subscript 1 the initial velocity.

$$\delta W = pdAds = pdV$$

Increase in potential energy $= mg(z_2 - z_1)$

Consider a fluid inside a deformable container. The



Force exerted by the pressure on an elemental area *dA* is *pdA*. If the elemental area **moves a distance *ds*** along the normal to the area, the work done by the force due to the pressure is

$$\delta W = pdAds = pdV$$

Process	Work transfer, ${}_1W_2$
Constant Volume	0
Constant pressure	$p(V_2 - V_1)$
$pV^n = c =$ $p_1V_1^n = p_2V_2^n$	$\dfrac{p_2V_2 - p_1V_1}{1-n}$
$pV = c =$ $p_1V_1 = p_2V_2$	$p_1V_1 \ln\dfrac{V_2}{V_1}$

PROPERTIES AND STATE

We will limit our discussion to simple compressible substances substances which are capable of only one way of doing reversible work on the surroundings - by expansion.

*Any two independent properties uniquely determine the other properties.*For example if we specify the temperature and pressure the specific volume is fixed and cannot be independently specified.

Ideal Gases:

$pv = RT$

Denoting the volume of a mole of gas by $\bar{V}$ we have the relation,

$p\bar{V} = \bar{M}RT = \bar{R}T \qquad R = \bar{R}/\bar{M}$

$\bar{R} = 8.314$ KJ/kmole K $= 1.545$ ft-lbf/lb mole R

$pV = n\bar{R}T = mRT \qquad R = \dfrac{\bar{R}}{\hat{M}}$

For a liquid, that changes phase,

Saturated Liquid	The liquid has just begin to evaporate
Saturated Vapor	All the liquid has just evaporated
Compressed Liquid	Liquid temp. below the saturation temp. at the same pressure
Superheated vapor	Vapor temperature is above the saturation temperature at the same pressure
Saturated Mixture	Both liquid and vapor coexist

Property	State
$v(p,T) < v_f(p)$	Compressed liquid
$v_f < v < v_g$	Saturated mixture
$v_g < v$	Superheated vapor
$p(T) < p_{sat}(T)$	Superheated vapor
$p(T) = p_{sat}(T)$	Saturated mixture
$p(T) > p_{sat}(T)$	Compressed liquid

Conservation of Energy:

For a system

$$ {}_1Q_2 + m\left(u_1 + \frac{V_1^2}{2} + gz_1\right) = {}_1W_2 + m\left(u_2 + \frac{V_2^2}{2} + gz_2\right) $$

$$ h = u + pv $$

$$ c_v = \left.\frac{\partial u}{\partial T}\right|_v $$

$$ c_p = \left.\frac{\partial h}{\partial T}\right|_p $$

In the two-phase region

$$x = \frac{(\mathrm{v} - \mathrm{v}_f)}{\mathrm{v}_{fg}} = \frac{u - u_f}{u_{fg}} = \frac{h - h_f}{h_{fg}} = \frac{s - s_f}{s_{fg}}$$

Control Volume

Conservation of mass:

$$\frac{\mathrm{d}m_{cv}}{\mathrm{d}t} + \sum_{out} \dot{m} - \sum_{in} \dot{m} = 0$$

Conservation of energy

$$\frac{\mathrm{d}E_{cv}}{\mathrm{d}t} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}\left(u + pv + \frac{\boldsymbol{V}^2}{2} + gz\right)$$

$$-\sum_e \dot{m}\left(u + pv + \frac{\boldsymbol{V}^2}{2} + gz\right) = 0$$

In steady state,

$$\dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}\left(h + \frac{\boldsymbol{V}^2}{2} + gz\right) - \sum_e \dot{m}\left(h + \frac{\boldsymbol{V}^2}{2} + gz\right) = 0$$

For a compressed liquid, $\rho \approx$ constant

$$u_l(p, T) \approx u_f(T)$$

$$h_l(p, T) = u_l(p, T) + pv_l = u_f(T) + (p - p_{sat} + p_{sat})v_f$$

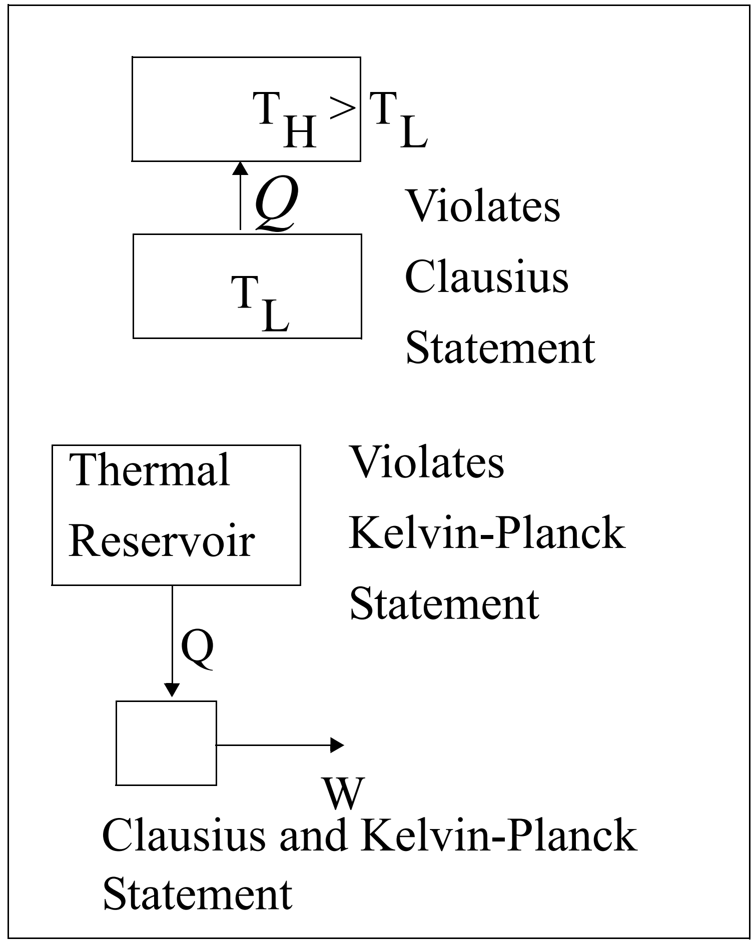
$$= h_f + (p - p_{sat})v_f$$

Second Law of Thermodynamics

The Clausius Statement: It is impossible for any system, operating in a cycle, to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body.

The *Kelvin-Planck* statement: *It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of work to its surroundings receiving energy by heat transfer from a single thermal reservoir.*

A reversible process is one which, having taken place in one direction, can take place in the reverse direction without leaving any trace of the processes having taken place, either on the system or on the surroundings.



Some of the causes of irreversibility are:

- Solid friction
- Viscous resistance of fluids
- Heat transfer with a finite temperature difference
- Any diffusion process such as mixing of different substances
- Sudden or unrestrained expansion
- Resistance heating
- Inelastic deformation
- . Hysteresis
- . Spontaneous chemical reaction

Second Law Corollaries:

Corollary 1: Operating between the same high temperature reservoirs (high temperature source and low temperature sink), no engine can be more efficient than the reversible engine.

Corollary 2: All reversible engines operating between the same source and sink have equal efficiencies.

Coefficient of Performance

For a refrigerator we define the *coefficient of performance* (instead of efficiency) β as

$$\beta = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$$

If it is operated as a heat pump, we define the coefficient of performance, γ as

$$\gamma = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$$

Clausius Inequality (Section 6.1)

The Clausius inequality states that when a material undergoes a cycle,

$$\oint \frac{\delta Q}{T} \le 0$$

Entropy Change

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T}\bigg|_{rev} \qquad s_2 - s_1 = \int_1^2 \frac{\delta Q/m}{T}\bigg|_{rev}$$

T-ds Equations

$$Tds = du + pdv$$

$$Tds = dh - vdp$$

Entropy Change of an Ideal Gas:

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

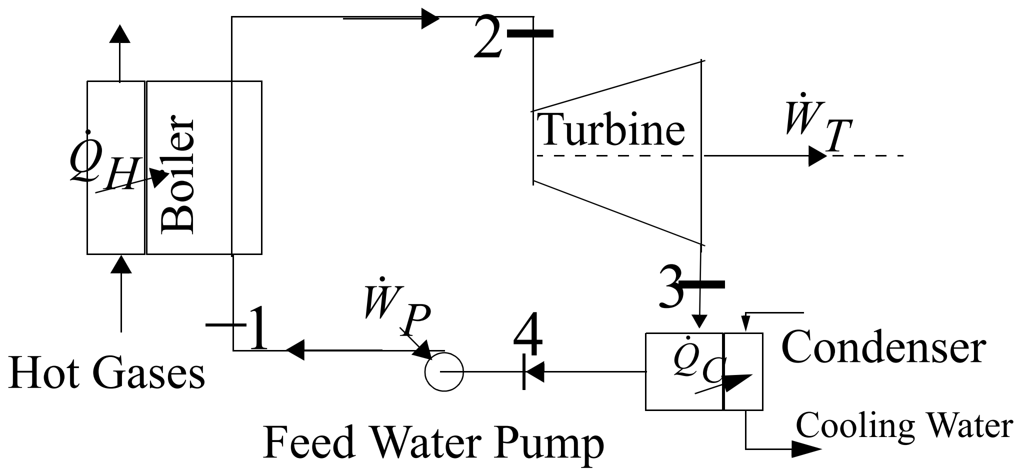
Entropy Change of an Incompressible Substance

$$s_2 - s_1 = c \ln \frac{T_2}{T_1}$$

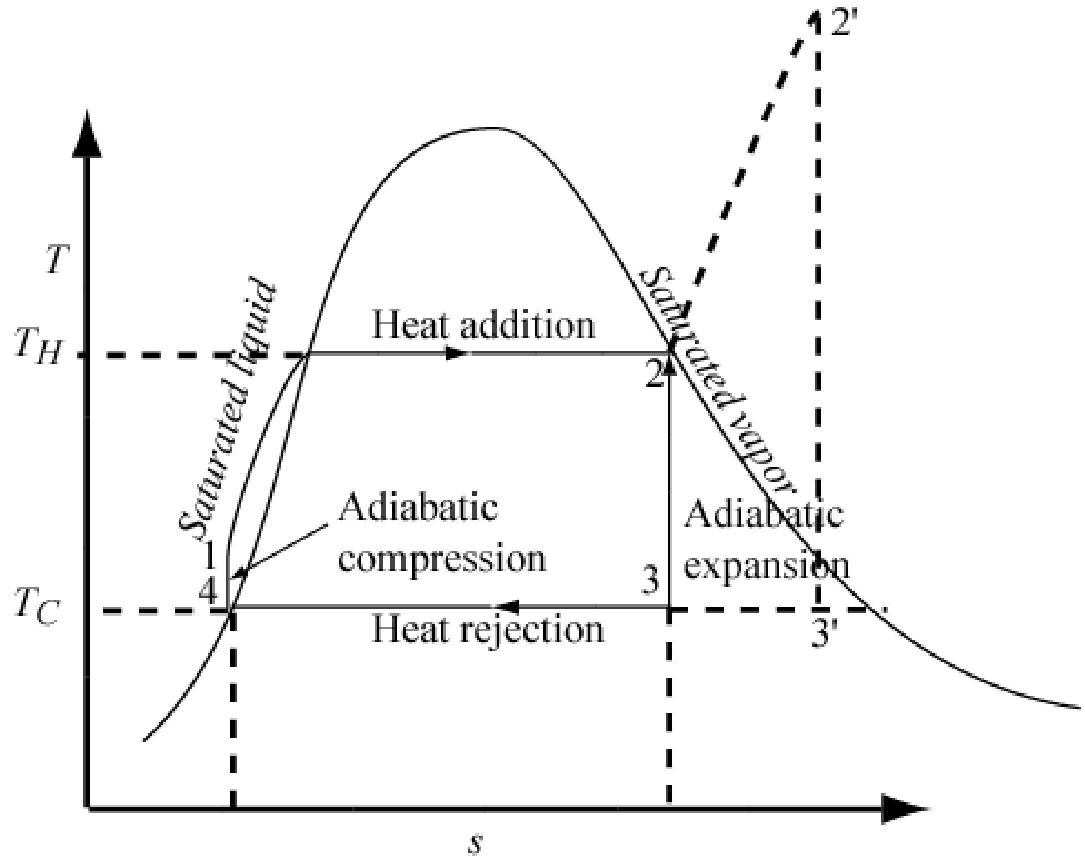
For an isentropic process of an ideal gas,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Steam Power Plant



Components of a Steam Power Plant



$$\dot{Q}_H = \dot{m}(h_2 - h_1)$$

$$\dot{W}_T = \dot{m}(h_2 - h_3)$$

$$\dot{Q}_C = \dot{m}(h_3 - h_4)$$

$$\dot{W}_p = \dot{m}(h_1 - h_4)$$

$$h_1 - h_4 = v_{f, p_{cond}}(p_1 - p_4)$$

$$\eta_{cycle} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_T - \dot{W}_p}{\dot{Q}_H}$$

Fluid Mechanics Review

Definition of a fluid

A substance that continuously deforms under the action of a shear force is a fluid.

Liquids can form a free surface in the presence of a gravitational field. *Gases* occupy the volume available.

Definition of fluid velocity

$$d\dot{m} = \rho \vec{V} \bullet \hat{n}\ dA \tag{1}$$

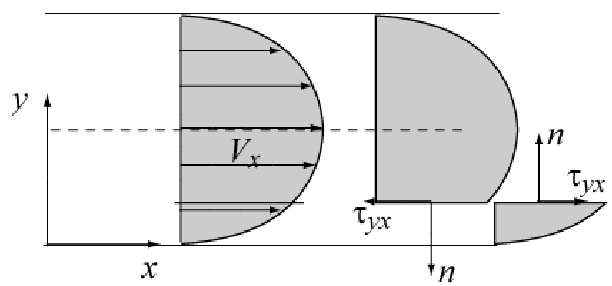
the dot product, $\vec{V} \bullet \hat{n}$, is the component of the velocity normal to dA. Thus an alternate form of Eq.(1) is

$$d\dot{m} = \rho\ V_n dA$$

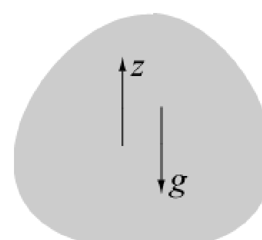
V_n = component of the velocity along the outward normal to dA.

Shear Stress

$$\tau_{yx} = \mu \frac{\mathrm{d}V_x}{\mathrm{d}y}$$



Pressure Variation in a Static Fluid



$$\frac{\partial p}{\partial z} = -\rho g$$

ρ = density of fluid

For ρ = constant,

$$p_2 - p_1 = -\rho g(z_2 - z_1) \tag{2}$$

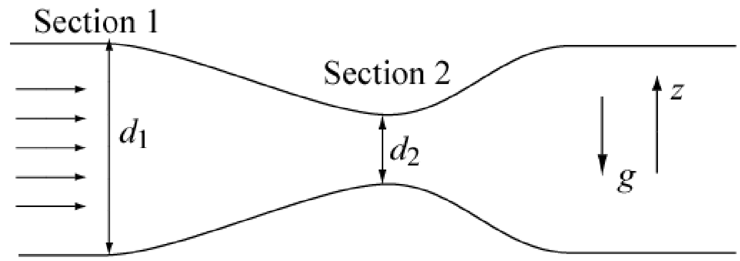
Bernoulli's Equation

In an inviscid, constant-density fluid in steady motion, for flow from 1 to 2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \tag{3}$$

Flow Measurement with a Venturimeter

Assuming that the elevation at points 1 and 2 are equal, from Eq.(3)



$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$V_2^2\left(1 - \frac{V_1^2}{V_1^2}\right) = 2\left(\frac{p_1 - p_2}{\rho}\right)$$

From mass conservation, $V_1 = V_2\left(\frac{d_2}{d_1}\right)^2 = V_2\beta^2 \qquad \beta = \frac{d_2}{d_1}$

$$V_2 = \left[\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}\right]^{1/2} \tag{4}$$

Conservation of Mass for a Control Volume - steady state, discrete number of inlets and exits.

$$\sum_{out} \dot{m}_i + \sum_{in} \dot{m}_i = 0 \tag{5}$$

Conservation of Linear Momentum - Control Volume - steady state

In any given direction, x,

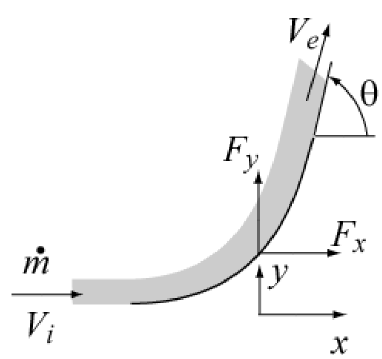
$$\sum F_x = \sum_{out} \dot{m}_i V_{xi} - \sum_{in} \dot{m}_i V_{xi} \tag{6}$$

F_x = Force exerted on the fluid in the CV in the positive x-direction.

For example, to find the forces exerted **by** the plate **on** the fluid,

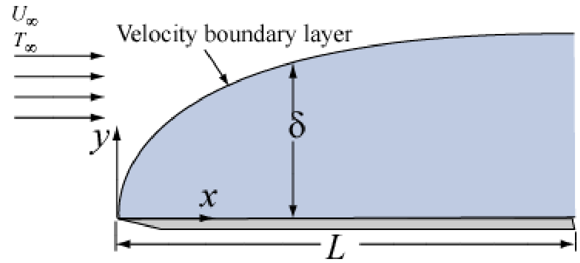
$$F_x = \dot{m}V_{ex} - \dot{m}V_{ix} = \dot{m}(V_e\cos\theta - V_i)$$

$$F_y = \dot{m}V_e\sin\theta$$



Flow Over a Flat Plate

Laminar flows



$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu} < 5\times 10^5$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

$$c_{fx} = \frac{\tau_{wx}}{(\rho U_\infty^2)/2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$c_{fL} = \frac{\overline{\tau_{wL}}}{(\rho U_\infty^2)/2} = \frac{1.328}{\sqrt{\mathrm{Re}_x}}$$

The total drag force on a flat plate is given by $F_D = \overline{\tau_{wL}}A$. Denoting the drag coefficient by C_D,

$$C_D = \frac{F_D}{A(\rho U_\infty^2)/2} = \frac{1.328}{\sqrt{\mathrm{Re}_x}}$$

For turbulent flow, $\mathrm{Re}_L > 5\times 10^5$,

$$C_D = \frac{0.031}{\mathrm{Re}_L^n} \qquad n = 0.114 - 0.2$$

Steady flow of an Incompressible Fluid in a Pipe from 1 to 2

Without any work transfer, dividing the energy equation by the mass flow rate, on a unit mass basis,

$$q + u_1 + \frac{p_1}{\rho} + \frac{\boldsymbol{V}_1^2}{2} + gz_1 = u_2 + \frac{p_2}{\rho} + \frac{\boldsymbol{V}_2^2}{2} + gz_2$$

In the equation, $u_2 - u_1 - q = h_f g$ where h_f represents the head loss due to viscous forces and other causes.

For laminar flows, in circular pipes of inner diameter d,

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\dot{m}}{\pi d \mu} < 2100$$

$$h_{flam} = f\frac{L}{d}\frac{V^2}{2g} \qquad f = \frac{64}{\mathrm{Re}_d}$$

For turbulent flows, $\mathrm{Re}_d > 2100$, the value of f depends on Re_d and on the roughness factor ε / d. For values of f for turbulent flows see the Moody chart.

For non-circular pipes, values of f for circular pipes can be used by defining the hydraulic mean diameter of the pipe as

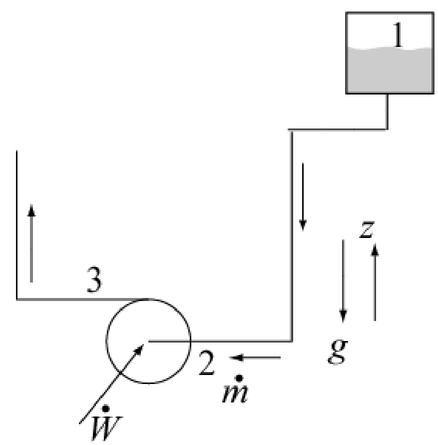
$$d_H = \frac{4 \text{ area of cross section}}{\text{wetted perimeter}}$$

Losses across bends, valves and so on are given by

$$h_{fminor} = K\frac{V^2}{2g}$$

Flow with a pump

$\dot{W} = \dot{m}(h_3 - h_2)$ where h is the specific enthalpy. Neglecting changes in the elevation in the pump



From *T-ds* equation for an incompressible fluid,

$$h_3 - h_2 = \mathrm{v}(p_3 - p_2)$$

The pump head (with the assumption of velocities at inlet and exit of the pump being equal) is defined as

$$h_p = \frac{p_3 - p_2}{\rho g}$$

Between 1 and 2 we have,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \qquad h_f = f\frac{L}{d}\frac{V^2}{2g}$$

If the reservoir is large, $V_1 \approx 0$.

For proper functioning of the pump, there should be no vapor at inlet; the pressure head at inlet should be greater than the vapor pressure head at inlet. The difference between the two is known as the Net Positive Suction Head (NPSH)

$$\text{NPSH} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} - \frac{p_v}{\rho g}$$