Forest Sampling
Avery & Burkhart, Chpt. 2&3

Reasons for Sampling
• Do NOT have the time or money to do a complete enumeration
• Remember that the estimates of the population parameters based on a sample are not accurate, therefore they include a certain amount of error
• Need to be able to quantify the amount of error

Scales of Measurement
• Different descriptive statistics are permissible depending on the scale used to measure to characteristic of a population

<table>
<thead>
<tr>
<th>Scale</th>
<th>Basic Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>Determination of equality</td>
<td>Assigning code number to tree species</td>
</tr>
<tr>
<td>Ordinal</td>
<td>Determination of greater or less (ranking)</td>
<td>Lumber or log grading, Site class estimation</td>
</tr>
<tr>
<td>Interval</td>
<td>Determination of the equality of intervals (arbitrary origin)</td>
<td>Temperature, Calendar time</td>
</tr>
<tr>
<td>Ratio</td>
<td>Determination of the equality of ratios (absolute origin)</td>
<td>Lengths, volumes, weights, Frequency of items</td>
</tr>
</tbody>
</table>

Sampling - General Approach
When dealing with the various approaches to sampling, we will be concerned with several parameters:
• \( \bar{y} \) combined with \( N \) =>
• \( s_{\bar{y}} \) combined with \( N \) =>
• CI of both \( \bar{y} \) and \( \hat{\bar{y}} \)
• \( n \) => required to achieve a specified level of accuracy in both \( \bar{y} \) and \( \hat{\bar{y}} \)

Sampling Error
Accuracy = \( f(\text{Precision, Bias}) \)
Sampling Error

Accuracy = f (Precision, Bias)

Bias - can control bias by applying proper probability sampling design
Precision - can increase precision (i.e., reduce error) by increasing sample size

Simple Random Sampling

• SRS is the most basic probability sampling method
• SRS forms the basis of all probability based sampling designs

Simple Random Sampling

The basic idea is SRS:
• in choosing a sample of \( n \) units, every possible combination of \( n \) units should have an equal chance of being selected
• this is not the same as requiring that every unit in the population has an equal chance of being selected
• the selection of any given unit should be completely independent of the selection of all other units

Simple Random Sampling

• Selection of a SRS from a population requires the development of a Sampling Frame (i.e., a list of all the sampling units in a population)
• Sample units are then randomly selected from the frame

Simple Random Sampling

• How does SRS control bias?
• Does increased sample size increase precision (reduce standard error of estimate)?
Developing a sampling frame

63 ACRE FOREST

sampling unit = 1/5 acre plot

Example (continued)

1. Determine average ft³ volume per plot and total ft³ volume in entire 63 acre forest stand

2. Randomly select ten plots from the population of 315 plots within the stand

Example (continued)

Mean

\[ \bar{y} = \frac{\sum y_i}{n} = \text{ft}^3/\text{plot} \]

Variance

\[ S_y^2 = \frac{\sum (y_i - \bar{y})^2}{n - 1} = (\text{ft}^3/\text{plot})^2 \]

Std. Dev.

\[ S_y = \sqrt{S_y^2} = \text{ft}^3/\text{plot} \]

SRS - Continuous variables

Mean:

\[ \bar{y} = \frac{\sum y_i}{n} \]

Standard Error of the Mean:

\[ S_y = \sqrt{\frac{S_y^2(N-n)}{n(N-n)}} \]

Variance:

\[ S_y^2 = \frac{\sum (y_i - \bar{y})^2}{n - 1} \]

Confidence Interval of the Mean:

\[ \bar{y} \pm tS_y \]

Example (continued)

Sample / Plot #  y
1  22  22.1
2  164  18.6
3  16  29.8
4  88  19.9
5  290  11.6
6  189  18.9
7  34  23.5
8  124  16.7
9  92  13.5
10  248  27.6

\[ \sum y_i = 10315 \]

\[ \sum y_i^2 = 22.1^2 + 18.6^2 + \ldots + 27.6^2 \]

\[ \bar{y} = 984.32 \]

\[ N = 315 \]

\[ s_y = \sqrt{32.984} = 5.75 \]

\[ n = 10 \]

\[ 10^2 = 100 \]

\[ \frac{315}{10} \]

\[ \frac{315}{10} \]

\[ \frac{315}{10} \]

\[ \frac{315}{10} \]

\[ 1.787 \]
Example (continued)

90% Confidence Interval
\[ t = \]
CI(factor) = \( t \times S_y \)
\[ = \]
\[ = \]
Lower bound = 20.22 - 
Upper bound = 20.22 + 

SRS - Continuous variables

Size of sample to achieve a specified error (E) about the mean:
\[ n = \frac{N \times t^2 S_y^2}{N \times E^2 + t^2 S_y^2} \]

Size of sample to achieve an allowable error (E%) about the mean:
\[ n = \frac{N \times t^2 CV^2}{N \times E%^2 + t^2 CV^2} \]

Example (continued)

Determine the number of sample plots to measure to attain a desired error of the estimation of mean volume of 1 ft³/plot at 95% confidence
\[ E = \]
\[ t = \]
\[ N = \]
\[ S_y^2 = \]

Solving for \( n \):
\[ n = \frac{315 \times 1.960^2 \times 32.984}{315 \times 1^2 + 1.960^2 \times 32.984} = 53.64 \]

SRS - Continuous variables

Standard Error of the Population Total:
\[ s_{\hat{y}} = N \times s_y \]
\[ \hat{T} = N \times \bar{y} \]
Confidence Interval of the Population Total:
\[ \hat{T} \pm t \times s_{\hat{y}} \]
Example (continued)

Total Volume in forest stand

\[ T = N \bar{y} \]

\[ = 315 \times 20.22 \]

\[ = 6369.3 \]

Stand Error of Total Volume

\[ s_T = N s_y \]

\[ = 315 \times 1.787 \]

\[ = 562.93 \]

95% Confidence Interval for Total

\[ T \pm t \times s_T \]

\[ 6369.3 \pm 2.262 \times 562.93 \]

(5095.9, 7642.7)

SRS - Continuous variables

Size of sample to achieve a specified error (E_T) about the Population Total:

\[
 n = \frac{N s_y^2}{E_T^2 + S_y^2} 
\]

SRS - Discrete variables

Restriction on the values for the y_i's

Simplest case, y is a binomial variable

e.g., \[ y_i = \begin{cases} 
1 & \text{if success} \\
0 & \text{if failure} 
\end{cases} \]

SRS - Discrete variables

Mean = Population Proportion:

\[
 \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{\text{number of successes}}{\text{total number observed}} = \hat{p} 
\]

SRS - Discrete variables

Variance:

\[
 S_y^2 = \frac{\sum y_i^2 - \left( \sum y_i \right)^2}{n} 
\]

\[
 = \frac{\sum y_i^2 - \left( \sum y_i \right)^2}{n^2 - 1} 
\]

\[
 = \frac{\sum y_i \left( \sum y_i \right)}{n^2 - 1} 
\]

SRS - Discrete variables

Standard error of mean:

\[
 S_\bar{y} = \sqrt{\frac{\sum y_i^2}{n} \left[ \frac{N - n}{N} \right]} 
\]

\[
 = \sqrt{\frac{\sum y_i \left[ \frac{N - n}{N} \right]}{n^2 - 1}} 
\]

\[
 S_\bar{y} = \sqrt{\frac{\hat{p}(1 - \hat{p}) \left[ \frac{N - n}{N} \right]}{N - 1}} 
\]