

#### Introduction

In country-level carbon accounting, as ecosystem studies, where using replication to establish confidence is not an option, other approaches are needed for quantifying uncertainty. Options for propagating uncertainty include Addition in Quadrature and Monte Carlo Simulation. However, it is easy to make mistakes, and incorrect implementation has been advised, for example, to reduce reported **Uncertainty in Country-Level Carbon Accounting**, which affects the payments made to countries for reducing emissions from deforestation and forest degradation. It is easy for even well educated researchers to make mistakes that inflate or reduce errors. We share errors we have made involving **Independence of Errors in Monte Carlo Simulation**.

#### **Uncertainty in Country-Level Carbon Accounting**

Deforestation and forest degradation are important sources of net carbon emissions to the atmosphere. Countries seeking payments for reducing emissions must report uncertainty in their estimates, with high uncertainties resulting in up to 15% reductions in payments. A review of submissions of forest reference levels submitted to the United Nations Framework Convention on Climate Change (37 countries) and the Forest Carbon Partnership Facility (18 countries) reveals multiple mistakes in error propagation. The most egregious is the practice of treating Monte Carlo iterations as samples, and dividing by the number of iterations, sometimes resulting in combined uncertainties smaller than any of the input uncertainties (the lowest combined uncertainty reported by a country was 1.3%). Guidelines should be more specific as to how to obtain uncertainties from Monte Carlo simulations, and tools should be developed to support efficient computation and proper application of error terms. Investments in emission reductions should be made with known confidence, correctly estimated, even if uncertainties are high.

One approach to error propagation makes use of the mathematical rule that the variance of a sum is the sum of the variances: squaring the SDs, adding them together, and taking the square root gives the SD of the sum, if the errors are independent. Summing in Quadrature verified by Monte Carlo Simulation:  $3^2 + 4^2 = 5^2$ 

Number Carlo ite summind with SD 100 10,000 1,000,00

One country reported rainforest emissions with an uncertainty of 19% and mesic forest had an uncertainty of 24%. The square root of this sum  $(19\%^* 18 \text{ M ha})^2 + (24\%^* 9 \text{ M ha})^2$ , divided by the sum of the areas, is 15%. If we treated the rainforest in 10 subcategories, each with an uncertainty of 19%, the combined uncertainty using this propagation rule would be 6%. For a million subcategories, the uncertainty would be 0.02%, and so on.



This unreasonable result is due to the assumption (unwarranted, in this case) that the sources of error are independent. For example, if all your savings were invested in ExxonMobil, the uncertainty in the value of your holdings would be higher than if you had a diverse portfolio of stocks, bonds, currencies, commodities, and hedge fund strategies, because these have somewhat independent risks. However, if you treated your 100 shares in ExxonMobil as independent, compared to a single holding of 100 shares, it would be absurd to claim that you reduced your uncertainty 10-fold. When forest parcels share uncertainties--for example, in tree allometry and wood density estimates--these need to be applied properly. See Independence of Errors.

# **Quantifying Uncertainty in Forest Carbon Accounting** to Improve REDD+ Mitigation Efforts

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### **Addition in Quadrature**

of Monte ations 2 terms of 3 & 4	SD of the sum, for multiple trials
	4.7, 5.5, 4.9, 4.9
	5.02, 5.015, 5.006,
0	5.0002, 4.998, 4.996,

You can verify this rule using Monte Carlo simulation. Randomly choose a number with a mean of 0 and a standard deviation of 3, and add it to a random number with a mean of 0 and a SD of 4, and record the sum. Do this multiple times. The average answer should be close to 0 The SD of the estimates should be close to 5. The more estimates you have, the closer the answer will be to

### **Monte Carlo Simulation**

In Monte Carlo error propagation, the contributing uncertainties are randomly sampled in each of many iterations of a calculation, and the distribution of these multiple estimates indicates the uncertainty of the estimate.

A common error in interpreting this output is to report the uncertainty in the mean or median of that distribution as an indicator of the uncertainty in the individual calculations. For example, for the country shown below, the uncertainty reported to the FCPF should have been >1000%. Instead, the uncertainty in the median of the Monte Carlo estimates was reported, which was very small (7%), because the number of estimates was very large (10,000).



It is incorrect to treat each of the estimates from iterations of a Monte Carlo simulation as if it were an observation and calculate standard error from the set of iterations. However, this approach has been recommended for reducing error estimates! (McMurray, 2017)

Instead, uncertainty should be obtained from the dispersion of the Monte Carlo iterations, for example as a 90% confidence interval. This gives the correct answer, with precision increasing with the number of iterations (see Table 1).

# **Mistakes Involving Independence of Errors**

It is important to recognize which uncertainty sources are independent across observations and which are shared. For example, random error in measurement of tree diameter is independent for each tree. A shared source of error is in allometric equations describing the relationship between tree diameter and mass. It would be a mistake to apply this source of uncertainty independently for each tree. In Monte Carlo simulation, the uncertainty in the model should be sampled only once for each iteration; if the model underestimates the average tree biomass, it should be underestimated for all the trees at once.

	Uncertainty in the prediction of an individual	Uncertainty in the prediction of the mean	Both	
requency of Monte Carlo outcomes 00001 00000 00001 000000				1 tree 0.002 ha
		allh		10 trees 0.02 ha
	Alle		allh.	30 trees 0.06 ha
			allh.	50 trees 0.1 ha
			allh	100 trees 0.2 ha
2500 - 2000 - 1500 - 1000 - 500 - 0 -		-dlh-	allh	1000 trees 2 ha
2500 - 2000 - 1500 - 1000 - 500 - 0 -	n	- Allh-	-dlb-	10000 trees 20 ha
	2000 3000 4000 5000 6000	2000 3000 4000 5000 6000 Leaf mass (Mg.ha <sup>-1</sup> )	2000 3000 4000 5000 6000	

It is important to conduct random sampling at the appropriate points in the simulation. Applying systematic errors independently to each observation results in an underestimate of the true uncertainty.

## Join QUEST -- or QUERCA !

QUEST (Quantifying Uncertainty in Ecosystem Studies) is a Research Coordination Network devoted to promoting and improving the practice of error propagation, funded by the National Science Foundation. We will continue as QUERCA (Quantifying Uncertainty Estimates Required for Carbon Accounting) to help countries involved in reducing emissions from deforestation and forest degradation (REDD+) for climate mitigation, funded by USFS International Programs (SilvaCarbon).

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Monte Carlo simulation was used to quantify uncertainty in the mass of sugar maple leaves, based on the allometric regression relating tree diameter to foliar mass of 14 trees at Hubbard Brook. Uncertainty in predicting individuals is not important with large numbers of trees. Uncertainty in the prediction of the mean, applied at each iteration to all trees, is always important. Including both is important for small numbers of trees.

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