Addition in Quadrature

One approach to error propagation makes use of the mathematical rule that the variance of a sum is the sum of the variances: squaring the SDs, adding them together, and taking the square root gives the SD of the sum, if the errors are independent.

Summing in Quadrature verified by Monte Carlo Simulation:

<table>
<thead>
<tr>
<th>Number of Monte Carlo iterations summing 2 terms with SD of 3 &amp; 4</th>
<th>SD of the sum for multiple trials</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4.7, 5.5, 4.9, 4.9...</td>
</tr>
<tr>
<td>5,000</td>
<td>5.002, 4.98, 4.99...</td>
</tr>
<tr>
<td>1,000,000</td>
<td>5.0002, 4.98, 4.99...</td>
</tr>
</tbody>
</table>

You can verify this rule using Monte Carlo simulation. Randomly choose a number with a mean of 0 and a standard deviation of 3, and add it to a random number with a mean of 0 and a SD of 4, and record the sum. Do this multiple times. The average answer should be close to 0. The SD of the estimates should be close to 5. The more estimates you have, the closer the answer will be to 5.

One country reported rainforest emissions with an uncertainty of 19% and mesic forest had an uncertainty of 24%. The square root of this sum (19% × 18 M ha)^2 + (24% × 9 M ha)^2, divided by the sum of the areas, is 15%. If we treated the rainforest in 10 subcategories, each with an uncertainty of 19%, the combined uncertainty using this propagation rule would be 6%. For a million subcategories, the uncertainty would be 0.02%, and so on.

Monte Carlo Simulation

In Monte Carlo error propagation, the contributing uncertainties are randomly sampled in each of many iterations of a calculation, and the distribution of these multiple estimates indicates the uncertainty of the estimate.

A common error in interpreting this output is to report the uncertainty in the mean or median of that distribution as an indicator of the uncertainty in the individual calculations. For example, for the country shown below, the uncertainty reported to the FCPF should have been >1000%. Instead, the uncertainty in the median of the Monte Carlo estimates was reported, which was very small (7%), because the number of estimates was very large (10,000).

Uncertainty in Country-Level Carbon Accounting

Deforestation and forest degradation are important sources of net carbon emissions to the atmosphere. Countries seeking payments for reducing emissions must report uncertainty in their estimates, with high uncertainties resulting in up to 15% reductions in payments. A review of submissions of forest reference levels submitted to the United Nations Framework Convention on Climate Change (37 countries) and the Forest Carbon Partnership Facility (18 countries) reveals multiple mistakes in error propagation. The most egregious is the practice of treating Monte Carlo iterations as samples, and dividing by the number of iterations, sometimes resulting in combined uncertainties smaller than any of the input uncertainties (the lowest combined uncertainty reported by a country was 1.3%). Guidelines should be more specific as to how to obtain uncertainties from Monte Carlo simulations, and tools should be developed to support efficient computation and proper application of error terms. Investments in emission reductions should be made with known confidence, correctly estimated, even if uncertainties are high.

Mistakes Involving Independence of Errors

It is important to recognize which uncertainty sources are independent across observations and which are shared. For example, random error in measurement of tree diameter is independent for each tree. A shared source of error is in allometric equations describing the relationship between tree diameter and mass. It would be a mistake to apply this source of uncertainty independently for each tree. In Monte Carlo simulation, the uncertainty in the model should be sampled only once for each iteration; if the model underestimates the average tree biomass, it should be underestimated for all the trees at once.

Join QUEST -- or QUERCA!

QUEST (Quantifying Uncertainty in Ecosystem Studies) is a Research Coordination Network devoted to promoting and improving the practice of error propagation, funded by the National Science Foundation. We will continue as QUERCA (Quantifying Uncertainty Estimates Required for Carbon Accounting) to help countries involved in reducing emissions from deforestation and forest degradation (REDD+) for climate mitigation, funded by USFS International Programs (SilvaCarbon).

- Find us at [www.quantifyinguncertainty.org](http://www.quantifyinguncertainty.org)
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**Quantifying Uncertainty in Forest Carbon Accounting to Improve REDD+ Mitigation Efforts**

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