COMPARISON OF METHODS FOR OPTIMIZING STRATIFIED SAMPLING ALLOCATION IN AREA ESTIMATION ACROSS MULTIPLE ESTIMATES

by

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A thesis
submitted in partial fulfillment
of the requirements for the
Master of Science Degree
State University of New York
College of Environmental Science and Forestry
Syracuse, New York
August 2024

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Acknowledgments

I want to express my deepest gratitude to everyone and every organization that has made this thesis possible and has been instrumental in this journey.

First and foremost, I sincerely thank my scholarship sponsors for their trust and for selecting me for their support. I am especially grateful to the Office of International Programs at the U.S. Forest Service (USFS) for their invaluable backing. My heartfelt thanks also go to the wonderful SilvaCarbon team: Craig, Oswaldo, Marcela, Paola, and Cath. Their collaboration and enthusiasm have been a constant source of inspiration and motivation. Their support was crucial for the completion of this project.

To my academic team, which I consider the best I could have had, I owe a special thanks to my first and foremost advisor, Dr. Stephen V. Stehman. His guidance and unwavering support, as well as the warmth and kindness of his family, made me feel at home throughout my time here. I also wish to express my gratitude to the members of the Forest Ecology Lab, Dr. Ruth Yanai, Dr. John Drake, and the rest of the Quantifying Uncertainty Estimates and Risk for Carbon Accounting (QUERCA) team. Their valuable contributions and camaraderie have deeply enriched my academic experience.

I am grateful to SUNY-ESF for the two years of academic and personal growth it provided. The education, the wisdom of the teachers, and the educational environment have been essential to my development and success. The quality and dedication of the faculty have left a lasting impact on my life.

My thanks to my great committee: Dr. Craig Wayson, Dr. Javier García Pérez, and Dr. Dingfan Xing. Their meticulous review and valuable suggestions were crucial for the improvement and completion of this thesis.

To my fellow adventurer, Ana V. Medina, I deeply appreciate your companionship and support throughout this journey. Your presence has made this experience much more enriching and meaningful.

Lastly, I want to express my deepest gratitude to my family, friends, and everyone from around the world and those I met along this adventure, who made this experience more manageable and beautiful. Your unconditional support and constant encouragement provided me with the emotional strength needed to achieve this milestone. Without you, this journey would not have been possible.

To all of you and, from the bottom of my heart, my most sincere thanks.

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ABSTRACT

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Area estimation is crucial in forest monitoring at local, national and global levels. Stratified sampling is widely used, with estimates derived from samples and satellite imagery. In programs like REDD+, map strata based on classes such as deforestation and afforestation may greatly improve precision of area estimates. Sample allocation to strata is critical, and Neyman allocation is optimal for a single target variable. This research examined the standard errors of different allocation methods, Average Optimal (AvgOpt), Bethel, and SSW, when multiple target estimates are involved. Evaluations used real populations and populations constructed from an experiment design that controlled reference proportions and map accuracy. Although no method was universally superior, the simple AvgOpt method performed comparably or better than Bethel and SSW in most populations. Bethel/SSW standard error ratios were sensitive to area disparities. AvgOpt/SSW ratios were correlated strongly with area differences and the percentage of the more common target class.

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1. INTRODUCTION

Area estimation plays a crucial role in forest monitoring, encompassing various local, national, and global applications. For example, accurate estimation of land surface allocation is indispensable for country-level reporting within initiatives such as the Reducing Emissions from Deforestation and Forest Degradation (REDD+) program (Olofsson et al., 2014). This includes precise measurement of activity data, which delineates human actions such as deforestation and degradation within forested areas (Aryal et al., 2021).

Accurate (i.e., unbiased) and precise estimation of these parameters representing forest loss, forest gain, and stable forest are essential for effective forest monitoring for several reasons. Firstly, it enables the identification of areas experiencing substantial changes, allowing for targeted conservation efforts and policy interventions. Secondly, it aids in assessing the overall health and resilience of forest ecosystems, guiding sustainable management practices. Thirdly, precise estimates are crucial for countries participating in initiatives like the REDD+ program, where results-based payments are linked to verifiable reductions in emissions. The meticulous collection and analysis of activity data, coupled with accurate and precise estimation of deforestation, degradation, and forest gain, serve as the cornerstone for informed and effective forest monitoring. These efforts play a pivotal role in addressing the challenges posed by climate change, ensuring the preservation of forests as vital components of the global ecosystem (Goetz et al., 2015).

The identified problem revolves around the challenges presented in achieving this precision improvement, given the spatial variability of forests and the complexities involved in sampling to assess changes in forest carbon. It is imperative to recognize the importance of area estimation and the role of sampling in overcoming the challenge of obtaining a census of reference data. Sampling becomes a critical aspect, and stratified sampling is often applied to the area estimation problem. Choosing an effective sample allocation to strata plays a fundamental role in ensuring the lowest possible standard error (i.e., best possible precision) for the estimated area, for example, in estimating levels of forest carbon. Sampling in scientific research and data collection provides an effective methodology for obtaining conclusions about a larger population that are supported by statistical inference. Sampling saves resources and time by selecting a subset of the population under study. Among the various types of sampling, stratified sampling emerges as an essential tool. This approach divides the population into homogeneous strata or subgroups potentially yielding improved precision relative to simple random sampling. Additionally, stratified sampling leverages geographic information through the use of maps or imagery to delineate strata, strategically allocating the sample based on specific spatial characteristics. This integration of mapping technology enhances the effectiveness of stratified sampling in addressing spatial variability (Olofsson et al., 2014; Stehman & Wagner, 2024; Wagner & Stehman, 2015).

Stratified random sampling is a practical approach, primarily focused in this study on area estimation. This sampling method offers flexibility by allowing an increased sample size for classes covering a small proportion of the area. This, in turn, reduces standard errors of area estimates for less common classes, aligning with the overarching goal of estimating area specific to each class. In terms of desirable design criteria, stratified random sampling stands out as a probability sampling design that is relatively straightforward to implement. It is a commonly employed method in accuracy assessment and area estimation, offering the advantage of being well-known within the remote sensing community (Olofsson et al., 2014).

Within stratified sampling, the allocation of the sample to different strata is a crucial phase that directly influences the precision of the estimates. The sample size allocated to each stratum is an important decision, and the allocation will depend on the objectives specified. Allocation refers to the proportion of the sample assigned to each stratum and is determined based on various criteria, such as the internal variability of the strata and the specific objectives of the research (Stehman & Wagner, 2024). There are different allocation methods, each with its advantages and practical considerations.

1. Optimal (Neyman) Allocation: Aims to minimize the variance of an estimate by assigning a larger sample size to strata with higher variability, and also taking into account the relative size of each stratum within the study region (Neyman, 1934). Neyman allocation applies to the case for which a single target estimate is the objective of the optimization.
2. Proportional Allocation: Distributes the sample proportionally to the size of the strata, ensuring that each stratum contributes according to its relative size (area in this application) in the total population; proportional allocation is an equal inclusion probability sampling design which translates into greater simplicity of analysis (i.e., sample observations do not need to be weighted as they would for an unequal probability sampling design). Proportional allocation provides a useful indicator of the effectiveness of strata. That is, both proportional allocation and simple random sampling are equal probability sampling designs for which all sample units have the same probability of being included in the sample. The precision of the area estimate for proportional allocation accounts for the stratification, whereas the precision of the estimate from simple random sampling does not. Because both sampling designs have the same equal inclusion probabilities, any difference in precision between simple

random sampling and proportional allocation is attributable to the effectiveness of the strata.

3. Equal Allocation: Assigns the same sample size to all strata, regardless of their size, which can simplify implementation but may not be efficient in terms of precision. Equal allocation is often used when the primary objective of sampling is assessing per class user's accuracy of a map. While not directly related to the area estimation objective, evaluating precision of equal allocation provides understanding of the trade-offs of allocating the sample for one set of objectives (estimating user's accuracy) versus an allocation targeting different objectives (estimating area).
4. Bethel Allocation: Determines the total sample size and allocation of the sample to strata for multivariate optimization (i.e., two or more estimates, for example forest loss and forest degradation). Similar to the Särndal et al. (1992) method that follows, the Bethel method extends the idea of Neyman allocation to two or more target estimates (Bethel, 1989). The main goal of this allocation is to minimize costs under the constraints of specified precision levels of estimates (coefficient of variation, CV) (De Meo, 2022). The Bethel allocation procedure is available in the R package 'SamplingStrata'.
5. Särndal et al. (1992, p. 469) (SSW) Allocation: This method is particularly useful in multivariate stratified surveys, where the goal is to work out a compromise allocation when the objective is to estimate area of two or more classes.

It is crucial to emphasize that both Bethel and SSW allocations play a significant role in optimizing survey estimates, particularly when dealing with multiple estimates. Unlike Neyman allocation, which focuses on optimizing a single estimate, the Bethel and SSW allocations are designed to enhance precision by considering two or more estimates simultaneously. Each allocation method has specific implications for the precision of the estimates, and it is crucial to select the appropriate approach based on the objectives and characteristics of a specific application.

Precision of area estimates is indispensable for well-informed decision making. The fundamental question that guides this research is how the different optimal allocation methods used in the stratified sampling approach compare in terms of reducing standard errors when more than one target estimate is of interest. The research objectives are the following: 1) evaluate and compare different optimal allocation methods to identify those that offer more precise estimates of area that will contribute better information to assessments; 2) assess the relationship between relative performance of the proposed allocation methods with factors such as the proportion of area of the different target classes and the accuracies of maps used to construct strata (e.g., user's and producer's accuracies). The second objective is primarily addressed by a designed experiment in which populations (cases) are created to assess the impacts of particular features of the populations.

2. METHODS

2.1. Area Estimation from an Error Matrix

The approach to area estimation addressed in this thesis connects directly to the error or confusion matrix of map accuracy assessment (Stehman, 2013). As indicated in Olofsson et al. (2014), the error matrix is a crucial component for assessing the accuracy of classifications based on remote sensing data. To create an error matrix, we compare classifications from remote sensing data (i.e., map labels) to reference labels (i.e., "reference" meaning the best available assessment of ground condition). A cross-tabulation is used to compare the class labels assigned by the classification with the reference labels obtained from sample sites. The matrix enables the quantification of both accuracy and area. Correct classifications are highlighted along the main diagonal of the matrix, while the off-diagonal areas indicate omission and commission errors. Omission errors occur when reference land change is observed but mapped as a stable class, potentially leading to significant uncertainty in the parameter estimates derived from the sample data, such as area estimates of land change (Olofsson et al, 2020). Commission errors occur when the classifier incorrectly assigns (maps) a pixel to a class, leading to an overestimation of the area for that class (Olofsson et al, 2013). The rows of the matrix correspond to the map labels derived from the classification of remote sensing data, whereas the columns correspond to the reference labels.

The error matrix is useful for estimating the area proportion of various classes. For example, if forest loss is a class of interest, the row total of the error matrix would show the area assigned as forest loss by the map, and the column total would show the area assigned to forest loss according to the reference classification. However, the proportion of area according to the reference classification must be estimated from the sample, which introduces uncertainty due to sampling variability (Olofsson et al., 2014). When using stratified random sampling, one of the most common approaches is a direct stratified estimator with the map classes defined as strata. This type of estimator is unbiased and often recommended in the literature for reducing standard errors (Stehman, 2013). Stratified sampling requires consideration of the sample allocation to strata, the subject of this research.

2.1.1. Evaluation of sample allocation options

We evaluated six allocation methods for stratified sampling. These methods included: 1) Optimal allocation (Neyman); 2) Average of Neyman allocations applied separately for each individual estimate; 3) Proportional allocation; 4) Equal allocation; 5) Bethel allocation, and 6) SSW. The reason for including each method is provided in the following. Neyman's optimal allocation is considered a standard approach, but it is designed to optimize a single target estimate. As stated earlier, proportional allocation is not intended to be optimal, but it does provide one way of quantifying the effectiveness of strata (i.e., comparing precision of proportional allocation, which has equal inclusion probabilities, to optimal allocation, which is the most effective allocation for the selected strata). Bethel and SSW allocations represent approaches that were designed to optimize simultaneously estimation of two or more parameters, and therefore these methods extend the concept of Neyman allocation to more complex situations. The average of the Neyman allocations from the optimization of each target parameter individually is an intuitively appealing and easy to compute allocation but is *ad hoc* in nature (i.e., not based on a formal optimization protocol). Although the "Equal" allocation method is less relevant to the area estimation objective, it is a common design for reference data collection for accuracy assessment and is included here to allow comparison to the allocations more targeted toward the objective of area estimation.

In the following steps, each allocation method mentioned above is described in detail. The methods require information that must be extracted from an error matrix. The stratum size expressed as a proportion of area of the stratum is $W_h = \frac{N_h}{N}$, and P_h is the proportion of the area of stratum h that is the target class according to the reference classification. For example, if the forest loss stratum has area of 20 km^2 and 10 km^2 of that stratum area is actual (reference) forest loss, P_h for that stratum is 0.50. For simple random sampling within strata and assuming all sample units within a stratum are the same size (area), the population proportion is

$$P_h = \frac{N_{jh}}{N_h} \quad (1)$$

where N_{jh} is the number of units with the reference label of the target class and N_h is the sample size in stratum h. The population variance in stratum h (for large N_h) is

$$P_h(1 - P_h) = \left(\frac{N_{jh}}{N_h}\right)\left(1 - \frac{N_{jh}}{N_h}\right) \quad (2)$$

Note that this variance corresponds to the usual population variance of a variable y, but here we have the special case that y=1 if the pixel has the reference class label of the target class and y=0 otherwise. The square root of the stratum variance (standard deviation) multiplied by the stratum weight (W_h) is an important input to the optimal allocation calculations:

$$W_h\sqrt{P_h(1 - P_h)} \quad (3)$$

The total sample size (n) is not critical to this analysis because the primary interest is the percent allocation of the sample to strata and this allocation is independent of n.

1) Optimal Allocation (Neyman)

$$n_h = n\frac{W_h\sqrt{P_h(1-P_h)}}{\sum_{h=1}^{H} W_h\sqrt{P_h(1-P_h)}} \quad (4)$$

2) Proportional Allocation

$$n_h = nW_h \quad (5)$$

3) Equal Allocation

$$n_h = \frac{n}{H} \quad (6)$$

where H is the number of strata.

4) Bethel

To determine the optimal allocation while meeting precision requirements across multiple variables (i.e., target estimates), we utilized the 'SamplingStrata' package in R. This package employs a genetic algorithm approach, treating each potential allocation as an individual within a population. The fitness of these individuals is assessed using the Bethel-Chromy algorithm, ensuring that resulting sample sizes meet specified precision constraints for two or more target estimates (De Meo, 2022). The functionalities provided by the 'SamplingStrata' package enable:

(a) Analysis of optimization results.

(b) Selection of a sample from the frame based on the optimal allocation.

The 'Bethel' function within the 'SamplingStrata' package determines the optimal sample size allocation under predefined precision constraints, for example by specifying the desired coefficient of variation (CV) for the target estimates. To implement the Bethel allocation, the input dataset should contain the following information:

1. Number of strata, H
2. Mean and standard deviation of each target variable within each stratum, which is obtained as $S_h = \sqrt{P_h(1 - P_h)}$
3. Population size (N_h) of each stratum.
4. Indication of whether the stratum undergoes a census (=1) or a sample (=0) (CENS); for this case we set CENS=0 to indicate a sample is used.
5. Value of the domain of interest to which the stratum belongs (DOM1), default suggested value used = 1 (i.e., we have just a single domain of interest, the entire “region of interest” that the area estimates are intended to apply to). A domain is also a subset or subgroup of a population, but unlike a stratum, a domain is not used in the selection of the sample. Domains are often subgroups of interest for reporting results. A subgroup can be both a stratum and a domain. This occurs when the subgroup is used in the sample selection and is also of interest for reporting results.

6. Cost associated with obtaining the data from a single unit in the stratum (COST), default suggested value used = 1 (i.e., equal cost per stratum).

In the analyses using the Bethel method, CV was specified to be equal for all target estimates and was set to 0.05. Changing the CV to a different magnitude would not change the percent allocation of the optimal allocation determined by the Bethel method. That is, as long as each class is specified to have the same target CV, whether that target is 0.05 or 0.20, the percent allocation of sample size to strata would be the same. The Bethel method computes the total sample size n needed to achieve the target CVs. The total sample size n determined by the Bethel procedure would be larger for the CV=0.05 case versus CV=0.20 because a larger n would be needed to achieve a smaller CV. Further, the achieved or realized CVs for the target estimates will not necessarily be the same even though equal CVs were specified as the input to the algorithm. The specified CV is the threshold that the optimization must attain, but it could achieve an even smaller CV for one or more of the target estimates included in the optimization. For example, if there are two target estimates and each is assigned a CV of 0.05 to attain in the optimization, the achieved CV of one of the classes may be smaller than 0.05 as the sample size increases to get the other class to the CV threshold of 0.05. Typically we would expect a rarer class to require larger n to achieve a CV of 0.05 and the more common target class would then have a CV smaller than 0.05.

<u>5) SSW</u>

The SSW allocation seeks to optimize the distribution of the sample between strata, taking into account the variability of the variables of interest within each stratum. This approach guarantees efficient sample allocation, maximizing the precision of population estimates. To calculate the SSW allocation, we implement the following steps using notation directly following Särndal et al. (1992, pp. 469-470):

$$V_{lin} = \sum_{i=I}^{I} H_i V_i \qquad (7)$$

where "V_{lin}" indicates that this variance is a linear combination of the variances of each target estimate, H_i is an importance weight assigned to the estimate (e.g., a larger weight could be assigned to estimating deforestation than to estimating degradation), V_i corresponds to the variance of each estimate i, and I is the number of strata. The sample size for each stratum is calculated such that

$$n_h \propto N_h \sqrt{\sum_{i=1}^{I} H_i S_{ih}^2} \qquad (8)$$

where S_{ih}^2 is the variance of the variable y_i in stratum h. Särndal et al. (1992) mention that the arbitrariness of the specified weights may be viewed unfavorably, but a similar concern could be expressed for choosing CV of the different estimates in the Bethel approach. In our analysis, we used equal weights (equal H_i) for the target classes.

6) <u>Average Optimal (AvgOpt)</u>

A variation of Neyman's Optimal Allocation, called 'Average Optimal,' was developed to extend the method to two or more target estimates. The technique involves computing Neyman's optimal allocation individually for each variable and then averaging these optimal allocations. This method has no formal mathematical support as optimal, but instead assumes that "splitting the difference" between the separate Neyman optimal allocations may prove to be a simple and effective allocation when two or more target estimates are of interest. For example, the average optimal allocation for two target estimates in one stratum h would be

$$n_h = \frac{n_{h1} + n_{h2}}{2} \qquad (9)$$

if n_{h1} is the Neyman optimal allocation for the first target class and n_{h2} is the Neyman optimal allocation for the second target class.

In the analyses that follow, the CV for the Bethel allocation was set to 0.05 for all target estimates and for the SSW allocation equal weights were specified. The two optimizations, therefore, are set up to achieve different objective functions and we would expect different outcomes from the two approaches. It is difficult to conceive of a way to equate the specification of CVs (Bethel) with specification of importance weights (SSW) that would create uniform initial conditions for the two optimization methods. In the case of optimizing to target two or more estimates, there are several ways to define optimization criteria so there is not a single solution to the problem. However, it would be useful to compare the outcomes of the different methods to better understand the differences in sample allocation and precision that would occur when applying these methods in different circumstances.

In practice, we could choose either the Bethel approach which operates on the basis of CV or the SSW approach which allows weighting of the target estimates based on specified importance weights of the estimates. In our analyses, we compare the performance of the Bethel and SSW optimizations under the simplest of conditions that could be adopted in practice which are equal CVs for Bethel and equal importance weights for SSW. Departures from equal CVs and equal importance weights would be application-specific so we do not explore here unequal CVs and unequal importance weights.

2.2. Experimental design

This component of the study focuses on how different characteristics of the population (i.e., error matrix) influence the allocation methods and resulting standard errors. Populations were created with three strata and two target classes while controlling factors such as user's and producer's accuracy. The aim is to comprehensively understand how these factors affect the precision of the area estimates. By creating populations with known characteristics, we are able to implement an experimental design approach because we control the levels of different factors potentially impacting performance of the different optimal allocation methods.

The experimental design consists of 32 populations each with H=3 strata and with two classes targeted for optimization. The factors evaluated in the experimental design are the following:

Factor 1: Reference proportions of the two target classes: Values of P_a and P_b were 0.01 and 0.05. (In terms of notation, the subscripts "a" and "b" are specific to the proportion of area of each target class, a and b. We also use "P" to denote stratum proportions and the subscript "h" with P_h represents the stratum number, h=1, …, H).

Factor 2: User's and producer's Accuracies: Values of 60% and 85% were used in all combinations within the two different classes, Class a and Class b, resulting in a total of 16 different combinations (see Table 1).

Factor 3: Omission error of each target class in the large W_3 stratum.

Table 1. Experimental design layout for populations with different combinations of user's and producer's accuracies for the two reference class proportions P_a and P_b.

Population	**Reference Proportion**		**Accuracy class a**		**Accuracy class b**	
	P_a	**P_b**	**User**	**Producer**	**User**	**Producer**
1	0.01	0.05	0.60	0.60	0.60	0.60
2	0.01	0.05	0.60	0.60	0.60	0.85
3	0.01	0.05	0.60	0.60	0.85	0.60
4	0.01	0.05	0.60	0.60	0.85	0.85
5	0.01	0.05	0.60	0.85	0.60	0.60

6	0.01	0.05	0.60	0.85	0.60	0.85
7	0.01	0.05	0.60	0.85	0.85	0.60
8	0.01	0.05	0.60	0.85	0.85	0.85
9	0.01	0.05	0.85	0.60	0.60	0.60
10	0.01	0.05	0.85	0.60	0.60	0.85
11	0.01	0.05	0.85	0.60	0.85	0.60
12	0.01	0.05	0.85	0.60	0.85	0.85
13	0.01	0.05	0.85	0.85	0.60	0.60
14	0.01	0.05	0.85	0.85	0.60	0.85
15	0.01	0.05	0.85	0.85	0.85	0.60
16	0.01	0.05	0.85	0.85	0.85	0.85

Two different populations were created for each of the 16 combinations of user's and producer's accuracies. These two populations, differentiated by the letters c and d (e.g., population 1c and 1d), had different P_h for classes a and b and were constructed to have different magnitudes of omission error to stratum 3, the stratum with the largest W_h. The c populations were constructed to have greater omission error in stratum 3, and the d populations were constructed to have smaller omission error in stratum 3. Construction of the 32 populations used in the designed experiment is described in the next section.

2.3. Construction of the Error Matrices in the Designed Experiment

The construction process is explained below using the first pair of populations, population 1c and 1d, as an example case.

Fixed Values (Green): These are known values and specified by the previously given P_a = 0.01 and P_b = 0.05, and the total of all cells in the error matrix which will always equal 1.

Table 2. Fixed values of constructed error matrices.

Population 1c		Reference			Total
		a	b	c	
Map strata	1				
	2				
	3				
Total		0.0100	0.0500		1.0000

Calculated Values (Purple): These values depend on our fixed values and so are calculated first. The value in cell a1 must be such that it yields the specified producer's accuracy of class a and so this cell is the product of the producer's accuracy for class a and P_a. Next we calculate the total for stratum 1 (cell Total 1) which is done so that the specified user's accuracy for class a is obtained. This is achieved by dividing the cell a1 value by the user's accuracy for class a.

Table 3. Calculated values of constructed error matrices.

Population 1c		Reference			Total
		a	b	c	
Map strata	1	0.0060			0.0100
	2				
	3				
Total		0.0100	0.0500		1.0000

Assigned Values (Yellow): Once the green cell and purple cell values are chosen, we need to specify two more cells of the error matrix (shaded yellow) and then the remaining cell values are determined by other constraints of the error matrix (i.e., row and column totals). For all the populations labeled as c, the values assigned for each class are 0.0001. By assigning this small value, most of the omission error for classes a and b is placed into the largest stratum which is stratum 3. For populations labeled as d, the goal was to place a small area of omission error for each of classes a and b into stratum 3. For class a, we chose the a2 cell so that the a3 cell was 0.0001, and for class b we chose the value of the 1b cell so that the 1c cell was 0.0001. It was not possible to choose the 1b cell to produce a value of 0.0001 in cell 3b (to match what was done for class a) while retaining the specified user's accuracy for class a.

Table 4. Assigned values of constructed error matrices.

Population 1c		**Reference**			**Total**
		a	**b**	**c**	
Map strata	**1**	0.0060	0.0001		0.0100
	2	0.0001			
	3				
Total		0.0100	0.0500		1.0000

Remaining Calculated Values (Blue): These values can be calculated after obtaining the previously described values. The first value to be calculated is in cell b2, which should be equal to the product of our P_b and the producer accuracy of class b. Next, the total for stratum 2 should be equal to the value obtained from the division of the value in cell b2 (previously calculated) by the user's accuracy of class b.

Table 5. Remaining values of constructed matrices.

Population 1c		**Reference**			**Total**
		a	**b**	**c**	
Map strata	**1**	0.0060	0.0001		0.0100
	2	0.0001	0.0300		0.0500
	3				
Total		0.0100	0.0500		1.0000

With this, the remaining values can be calculated simply by subtracting the known values from the total values.

Table 6. Constructed matrices of population 1c.

Population 1c		Reference			Total
		a	b	c	
Map strata	1	0.0060	0.0001	0.0039	0.0100
	2	0.0001	0.0300	0.0199	0.0500
	3	0.0039	0.0199	0.9162	0.9400
Total		0.0100	0.0500	0.9400	1.0000

For population 1d, the process is the same, except the assigned values (in yellow) are replaced by the largest values that allow us to create the matrix, which are directly obtained from the previously constructed matrix and are observed in cells c1 and a3 (0.0039).

Table 7. Constructed matrices of population 1d.

Population 1d		Reference			Total
		a	b	c	
Map strata	1	0.0060	0.0039	0.0001	0.0100
	2	0.0039	0.0300	0.0161	0.0500
	3	0.0001	0.0161	0.9238	0.9400
Total		0.0100	0.0500	0.9400	1.0000

2.4. Real populations

The data for this part of the research (Table 8) were obtained from the relevant scientific literature by selecting case study examples for which the source reported a full error matrix and included two or more rare classes that would be of interest as the target classes for optimization. These sources collectively offer a diverse range of methodologies, approaches, and findings related to land change detection and accuracy assessment. For our study, we extracted the relevant information from the confusion matrices and input that data into the different optimization methods.

More than one error matrix was obtained from some sources. This was because in some cases, more than two rare classes were identified in the error matrix, allowing for the analysis of various combinations of these classes. Additionally, some sources provided multiple error matrices, each with different accuracy values. The following were sources of data for the real populations:

Table 8. Sources of real populations and attributes of the target classes for optimizing area estimates.

Number	Source	Target Class	
		a	b
R1	Badjana et al. (2017) Table 4 RF Classification	Forest Loss	Savannah Loss
R2	Badjana et al. (2017) Table 4 SVM Classification	Forest Loss	Savannah Loss
R3	Wickham et al. (2023) Table 6	Barren Land	Wetland
R4	Chen et al. (2023) Table 1	New Plantation	Deforestation
R5	Yang et al. (2022) Table 1	Cover Change	Condition Change
R6	Chen et al. (2021) Table 7	Degradation Regeneration	Degradation no Regeneration
R7	Forest Reference Level Sudan (2020) Table 10	Gain	Loss
R8	Activity Data Report for the ER Program of Lao PDR (2018) Table 11	Degradation	Deforestation
R9	Wickham et al. (2023) Table 6	Water	Wetland
R10	Dymon et al. (2011) Table 3	Deforestation	Afforestation
R11	Chen et al. (2021) Table 8	Deforestation	Degradation
R12	Olofsson et al. (2014) Table 6	Forest Gain	Deforestation
R13	Chen et al. (2021) Table 7	Deforestation	Degradation no Regeneration
R14	Wickham et al. (2023) Table 6	Barren Land	Water
R15	Forest Reference Emission Level and Forest Reference Level Thailand (2020) Table 28	Forest Gain	Forest Loss
R16	The submission of Bangladesh’s Forest Reference Level for REDD+ under the UNFCCC (2018) Table 15	Reforestation	Deforestation
R17	The submission of Bangladesh’s Forest Reference Level for REDD+ under the UNFCCC (2018) Table 15	Degradation Low	Deforestation

The characteristics of the real populations are presented in Table 9. The number of strata *H* for each case study population is shown, along with the percentage of area and the user's, producer's, and overall accuracies for each target class. Class a is designated as the rarer (smaller) class, while class b is the larger of the two target classes (while still being one of the rare classes in each population).

For class a, the percent area ranged from 0.12% (R5) to 2.39% (R7) and for class b the values ranged from 0.62% (R5) to 9.54% (R1). These values reflect the variability in area distribution among different sources, with some sources showing substantially greater area coverage for certain classes compared to others.

Table 9. Characteristics of real populations.

Source	Strata	% Area		Accuracy %				
				Producer's		User's		Overall
		a	b	a	b	a	b	
R1	8	1.05	9.54	38.0	81.0	50.0	74.0	79.0
R2	8	2.05	8.54	88.0	40.0	33.0	49.0	67.0
R3	8	0.82	4.50	80.0	82.0	64.0	59.0	83.1
R4	5	0.75	4.81	56.2	48.0	100.0	46.8	84.5
R5	3	0.12	0.62	81.8	96.7	64.3	79.7	99.8
R6	7	1.94	3.02	86.0	28.8	37.0	64.3	88.9
R7	4	2.39	4.23	10.6	24.3	9.9	18.9	83.0
R8	6	1.62	3.13	69.2	86.7	60.0	86.7	92.3
R9	8	1.82	4.50	96.0	82.0	97.0	59.0	83.1
R10	4	0.29	2.17	93.8	97.1	93.4	97.1	99.6
R11	6	0.34	5.02	70.9	82.7	51.7	69.2	90.5
R12	4	1.12	2.01	81.8	70.0	60.0	70.0	95.0
R13	7	0.35	3.02	70.9	28.8	51.7	64.3	88.9
R14	8	0.82	1.82	80.0	96.0	64.0	97.0	83.1
R15	4	0.30	1.81	32.0	40.0	37.0	51.0	94.4
R16	8	0.83	1.00	65.2	93.9	40.0	41.3	68.7
R17	8	0.80	1.00	47.2	93.9	33.3	41.3	68.7
Minimum	**3**	**0.12**	**0.62**	**10.6**	**24.3**	**9.9**	**18.9**	**67.0**
Maximum	**8**	**2.39**	**9.54**	**96.0**	**97.1**	**100.0**	**97.1**	**99.8**

2.5. Ratios of Standard Errors of Area Estimates Under Different Optimizations

To simplify the results and be able to observe the advantage of one allocation method over another, ratios of the standard errors resulting from the different allocation methods were used. These ratios, which always are based on each allocation method having the same total sample size, allow comparison of the relative precision of the methods used. The following standard error ratios are reported in the comparisons as these three ratios compare the allocation methods specifically constructed to optimize two or more target classes:

1. Bethel/SSW
2. AvgOpt/Bethel
3. AvgOpt/SSW

Ratios smaller than one would favor the allocation method listed as the numerator of the ratio.

2.6. Correlation and Regression Analysis

Correlation and regression analyses were conducted to explore the relationships between population characteristics and the performance of the different allocation methods in stratified sampling to identify the strength and direction of these associations. These analyses included data on several key variables, including producer's, user's and overall accuracies, the percent area of classes a and b, and the differences between these class areas. We also looked at the ratio of the standard deviations of the two target classes, $\sqrt{[P_a(1-P_a)]/[P_b(1-P_b)]}$.

The primary focus was on evaluating associations between these factors and the standard error ratios Bethel/SSW, AvgOpt/Bethel and AvgOpt/SSW.

3. RESULTS

3.1. Sample allocation

Tables 10 - 12 show three examples of the sample size allocations resulting from the different methods (allocations for all populations can be found in the appendix). The total sample size is n=10,000 for all cases (the percent allocation of the sample to strata is obtained by inserting a decimal 2 places from the right of the sample size shown). In Table 10, the sample size allocations for the different methods are shown for real population R1 (see Table 9 for description of real populations). There is an advantage of smaller standard error in both classes when using the Average Optimal, Bethel, and SSW allocation methods compared to Simple Random Sampling (SRS). However, for the Savannah Loss class, no advantage is observed when using the Bethel method. The sample size allocation is very different among the different allocation methods. The Neyman optimal allocations for the two target classes are notably different in strata 1, 2, 5, and 6, and both Neyman allocations are substantially different from proportional allocation (e.g., stratum 1). In this example, the Bethel and SSW allocations differ greatly in strata 1, 2, and 3. However, the Bethel allocation is not very different from the Forest Loss Neyman allocation and this similarity is reflected in the closeness of the resulting standard errors for estimating Forest Loss (0.064 for Neyman optimal, 0.067 for Bethel).

Table 10. Sample allocations and standard errors for real population R1.

Stratum	P_h		Optimal			Propor tional	Equal	Bethel	SSW	
	Forest Loss	Savannah Loss	Neyman Forest Loss	Neyman Savanna h Loss	Avg Opt					
1	0.0252	0.0005	4872	281	2577	1984	1250	4974	1629	
2	0.0003	0.0335	976	4412	2694	3882	1250	1053	3635	
3	0.0037	0.0146	2585	2073	2329	2732	1250	2798	1904	
4	0.0087	0.0087	167	67	117	115	1250	183	78	
5	0.0588	0.0588	63	25	44	17	1250	43	29	
6	0.4819	0.1205	650	171	410	83	1250	208	257	
7	0.0010	0.7383	506	2897	1701	1043	1250	547	2383	
8	0.0075	0.0075	181	73	127	134	1250	196	85	
		Total	*10000*	*10000*	*10000*	*9990*	*10000*	*10000*	*10000*	**SRS**
		SE (Forest Loss)	**0.064**	**0.193**	**0.076**	**0.092**	**0.102**	**0.067**	**0.093**	**0.102**
		SE (Savannah Loss)	**0.310**	**0.158**	**0.189**	**0.195**	**0.254**	**0.299**	**0.169**	**0.294**

Table 11 shows the results of our sample size for the different allocation methods used for the real population R17. Both classes a and b had smaller standard errors when using the Optimal, Bethel, or SSW allocation methods compared to the standard errors of SRS. The sample size distribution was very different for the various allocation methods. The two optimal Neyman allocations were notably different in several strata (4, 5, 7). In this example, Bethel and SSW allocations differed substantially in strata 1, 2, 3, and 4. The Bethel allocation was not very different from the Neyman allocation for the rarer class Degradation Low, except for strata 1, 2, and 3, which may be reflected in the resulting standard errors (0.058 for optimal Neyman, 0.070 for Bethel). On the other hand, for Deforestation the Bethel allocation differed substantially from the Neyman allocation in most strata and the result of this difference was a much smaller standard error of 0.029 for optimal Neyman versus a standard error of 0.065 for the Bethel allocation. The SSW allocation was not very different from the Neyman allocation in both target classes and the near similarity in sample allocation was reflected in the closeness of the standard errors.

Table 11. Sample allocations and standard errors for real population R17.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Degradation Low	Deforestation	Neyman Degradation Low	Neyman Deforestation	AvgOpt					
1	0.0400	0.4133	711	3557	2134	211	1250	349	1567	
2	0.2494	0.0324	229	187	208	31	1250	51	202	
3	0.3277	0.0084	958	371	665	119	1250	197	796	
4	0.1607	0.1607	39	78	59	6	1250	10	45	
5	0.0533	0.0133	41	41	41	11	1250	18	37	
6	0.0019	0.0132	397	2091	1244	536	1250	464	915	
7	0.0023	0.0001	7402	3228	5315	8932	1250	8649	6179	
8	0.0058	0.0058	224	447	335	172	1250	263	259	
		Total	*10000*	*10000*	*10000*	*10018*	*10000*	*10000*	*10000*	**SRS**
		SE (Degradation Low)	**0.058**	**0.083**	**0.065**	**0.081**	**0.124**	**0.070**	**0.061**	**0.089**
		SE (Deforestation)	**0.052**	**0.029**	**0.033**	**0.080**	**0.044**	**0.065**	**0.037**	**0.099**

Table 12 shows the results of different allocation methods for real population R5. Population R5 is an interesting case because both classes comprise less than 1% of the area. However, the strata are very beneficial because the standard errors for proportional allocation were substantially smaller than the standard errors for SRS. Further, the Average Optimal, Bethel, and SSW allocations yield nearly the same standard errors with each other but had substantially smaller standard errors than proportional allocation and SRS. The two separate Neyman allocations differed by approximately 10% in each of strata 2 and 3. For example, in stratum 2, 10.15% was optimal for the class Coverage Change, while 22.16% was optimal for the Change of Condition class. The Bethel and SSW allocations were different by about 7% in strata 2 and 3, and the Average Optimal allocation was almost the same as that of Bethel and SSW. The great similarity in these assignments resulted in the standard errors of the optimal methods being very close to each other.

Table 12. Sample allocations and standard errors for real population R5.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Cover Change	Condition Change	Cover Change	Condition Change	AvgOpt					
1	0.6429	0.1429	567	365	466	14	3333	420	458	
2	0.0270	0.7973	1015	2216	1616	74	3333	1031	1770	
3	0.0001	0.0001	8418	7419	7918	9915	3333	8550	7771	
		Total	*10000*	*10000*	*10000*	*10003*	*10000*	*10000*	*10000*	**SRS**
		SE (Cover Change)	**0.0012**	**0.0012**	**0.0012**	**0.0025**	**0.0017**	**0.0012**	**0.0012**	**0.0035**
		SE (Condition Change)	**0.0014**	**0.0013**	**0.0014**	**0.0038**	**0.0018**	**0.0014**	**0.0014**	**0.0078**

3.2. Real populations - comparison of standard error ratios for different allocations

The Bethel method (Table 13) provides the total sample size n needed to achieve the CV values specified for the estimates targeted by the optimization. There is notable variability in this required sample size (n) for each population. The target Coefficient of Variation (CV) for the Bethel allocation was 0.05 for both classes a and b. Under this condition of equal CV, the Bethel allocation was strongly influenced towards the smaller (rarer) class. This influence was evident from the result that the actual CVs achieved by the Bethel allocation were 0.05 (the target CV) for class a (with one exception, the R10 case) whereas the CVs for class b were often smaller than 0.05. This suggests that achieving a CV of 0.05 for the rarer class was the primary driver (i.e., the more difficult requirement) for the optimization. For the Bethel allocation, the standard errors (SE) tended to be smaller for class a compared to class b when the same CV of 0.05 was assigned to both target classes.

Table 13. Bethel allocation CVs and SE.

Source (Population)	# Strata	% Area		Bethel					Target SE% for Bethel when CV=0.05	
				CV (n=10,000)		n	Actual CV (required n)			
		a	b	a	b		a	b	a	b
R1	8	1.05	9.54	0.06	0.03	14,248	0.050	0.027	0.05	0.48
R2	8	2.05	8.54	0.04	0.03	3,996	0.050	0.050	0.10	0.43
R3	8	0.82	4.50	0.06	0.04	9,884	0.050	0.050	0.04	0.23
R4	5	0.75	4.81	0.07	0.04	16,864	0.050	0.028	0.04	0.24
R5	3	0.12	0.62	0.01	0.002	33,362	0.050	0.011	0.01	0.03
R6	7	1.94	3.02	0.03	0.03	3,153	0.050	0.050	0.10	0.15
R7	4	2.39	4.23	0.06	0.05	14,632	0.050	0.037	0.12	0.21
R8	6	1.62	3.13	0.06	0.02	14,091	0.050	0.017	0.08	0.16
R9	8	1.82	4.50	0.02	0.02	2,310	0.050	0.050	0.09	0.23
R10	4	0.29	2.17	0.07	0.02	13,838	0.001	0.050	0.01	0.11
R11	6	0.34	5.02	0.10	0.02	17,926	0.050	0.017	0.02	0.25
R12	4	1.12	2.01	0.05	0.04	8,533	0.050	0.050	0.06	0.10
R13	7	0.35	3.02	0.10	0.03	18,217	0.050	0.025	0.02	0.15
R14	8	0.82	1.82	0.06	0.03	9,835	0.050	0.030	0.04	0.09
R15	4	0.30	1.81	0.17	0.07	78,941	0.050	0.023	0.02	0.09
R16	8	0.83	1.00	0.04	0.03	4,817	0.050	0.050	0.04	0.05
R17	8	0.80	1.00	0.09	0.07	20,775	0.050	0.035	0.04	0.05

"Actual CV" is the CV for each target class for the sample size n required to achieve a CV of 0.05 or smaller, and CV (n=10,000) is the CV for the benchmark sample size used to compare allocation methods.

In Table 14, the ratios of the SEs obtained from the Bethel, SSW, and Average Optimal methods are presented. Ratios less than or equal to 0.90 are highlighted in red and those greater than or equal to 1.10 are highlighted in yellow to indicate an advantage of at least 10% for one of the allocation methods. For example, considering the Bethel/SSW ratio, any cell in red indicates that the Bethel allocation performed at least 10% better than SSW (i.e., ratio less than 0.90) while any cell in yellow indicates that SSW had an advantage of at least 10% over Bethel (i.e., ratio greater than 1.10).

For the Bethel/SSW ratio and class a, Bethel performed substantially better than SSW for only the first three populations, and there was only one case for class a where SSW performed better than Bethel. However, for class b, nine cases showed substantially better performance when using SSW compared to the Bethel allocation method. This indicates that SSW generally would be preferred over Bethel when estimating the larger class b area.

For the ratios of standard errors for the Average Optimal and Bethel allocation methods, Bethel performed substantially better than AvgOpt in only one population in class a. Overall, there was often a substantial advantage to AvgOpt for both classes (a and b) compared to Bethel. In the comparison of AvgOpt with SSW, AvgOpt was substantially better in about half of the populations for class a and substantially better in only one case for class b. On the other hand, SSW had a large advantage over AvgOpt in three cases, all of which were in class b (the same three cases also observed as favorable to SSW in the Bethel/SSW ratio). As a general conclusion from these results, AvgOpt was preferable to Bethel when considering both classes a and b, whereas AvgOpt was preferable to SSW for class a and performed about the same as SSW for class b.

Table 14. Ratios of standard errors obtained from the different allocation methods for real populations.

Source	# Strata	% Area		Ratio					
				Bethel/SSW		AvgOpt/Bethel		AvgOpt/SSW	
		a	b	a	b	a	b	a	b
R1	8	1.05	9.54	0.72	1.76	1.14	0.63	0.82	1.11
R2	8	2.05	8.54	0.81	1.08	0.95	1.05	0.77	1.13
R3	8	0.82	4.50	0.84	1.54	0.98	0.72	0.83	1.12
R4	5	0.75	4.81	0.95	1.15	1.01	0.89	0.96	1.02
R5	3	0.12	0.62	0.98	1.07	1.01	0.94	0.99	1.01
R6	7	1.94	3.02	1.01	1.00	0.97	1.02	0.98	1.02
R7	4	2.39	4.23	1.02	0.99	0.98	1.00	1.00	1.00
R8	6	1.62	3.13	1.02	1.19	1.00	0.81	1.02	0.96
R9	8	1.82	4.50	1.05	1.00	0.81	1.05	0.86	1.05
R10	4	0.29	2.17	1.05	1.11	0.92	0.98	0.97	1.09
R11	6	0.34	5.02	1.05	1.28	0.77	0.85	0.81	1.09
R12	4	1.12	2.01	1.08	1.02	0.89	1.01	0.96	1.03
R13	7	0.35	3.02	1.08	1.29	0.78	0.84	0.84	1.07
R14	8	0.82	1.82	1.08	1.53	0.94	0.64	1.01	0.98
R15	4	0.30	1.81	1.05	1.07	0.91	0.95	0.96	1.02
R16	8	0.83	1.00	1.08	1.10	0.94	0.91	1.01	0.99
R17	8	0.80	1.00	1.14	1.78	0.94	0.50	1.06	0.89

3.3. *Experimental design populations - comparison of standard error ratios for different allocations*

From the descriptive statistics of the results of the standard error ratios from the populations of the designed experiment (Table 15), the mean value for the Bethel/SSW ratio in class b was 1.11 indicating that the SSW allocation had an 11% advantage over Bethel in class b. The mean value for the AvgOpt/Bethel ratio in class a was 0.88, indicating that the AvgOpt had a 12% advantage over Bethel in class a. The mean value for the AvgOpt/SSW ratio in class a was also 0.88.

Table 15. Summary statistics of the standard error ratios for the 32 populations of the designed experiment.

Ratio	Class	Mean	StDev	Minimum	Median	Maximum
Bethel/SSW	a	1.01	0.06	0.88	1.00	1.11
	b	1.11	0.16	0.99	1.04	1.60
AvgOpt/Bethel	a	0.88	0.13	0.62	0.92	1.03
	b	0.97	0.13	0.66	0.99	1.12
AvgOpt/SSW	a	0.88	0.10	0.67	0.91	0.98
	b	1.05	0.03	1.01	1.04	1.12

Similar to the results for the real populations, for the Bethel method (Table 16) there was substantial variability in the sample size required to achieve the target CVs. For the designed experiment, the populations labeled as c always required a larger sample size compared to those labeled as d. Additionally, populations with higher user's and producer's accuracy had a smaller required total sample size compared to those populations with lower accuracies. The Bethel allocation still showed a strong tendency towards the Neyman allocation of the smaller class a, except in 6 cases. These exceptions were all from the populations of the d type which were constructed to have a greater proportion of area in stratum 3.

Table 16. Bethel allocation results (Coefficients of variation) for the experimental design populations.

Population	Bethel				
	CV (n=10,000)		n	Actual CV (Bethel = n)	
	a	b		a	b
1c	0.074	0.038	6,179	0.050	0.033
1d	0.044	0.030	2,473	0.043	0.050
2c	0.074	0.038	6,126	0.050	0.037
2d	0.032	0.020	1,205	0.050	0.050
3c	0.074	0.032	6,173	0.050	0.026
3d	0.044	0.028	2,317	0.050	0.050
4c	0.074	0.026	6,179	0.050	0.023
4d	0.030	0.018	1,032	0.050	0.050
5c	0.053	0.035	3,815	0.050	0.044
5d	0.039	0.029	2,335	0.042	0.050
6c	0.053	0.032	3,841	0.050	0.041
6d	0.028	0.017	1,095	0.050	0.041
7c	0.053	0.031	3,791	0.050	0.039
7d	0.039	0.027	2,178	0.048	0.050
8c	0.053	0.023	3,815	0.050	0.029
8d	0.026	0.014	949	0.050	0.041
9c	0.067	0.038	6,146	0.050	0.033
9d	0.041	0.032	2,711	0.040	0.050
10c	0.067	0.038	6,093	0.050	0.037
10d	0.033	0.023	1,529	0.050	0.050
11c	0.067	0.032	6,140	0.050	0.026
11d	0.042	0.030	2,561	0.050	0.050
12c	0.067	0.026	6,146	0.050	0.023
12d	0.031	0.020	1,337	0.050	0.050
13c	0.045	0.035	3,805	0.050	0.044
13d	0.034	0.032	2,689	0.039	0.050
14c	0.045	0.032	3,820	0.050	0.041
14d	0.028	0.023	1,474	0.050	0.050
15c	0.045	0.031	3,781	0.050	0.039
15d	0.035	0.030	2,534	0.045	0.050
16c	0.045	0.023	3,805	0.050	0.029
16d	0.026	0.020	1,255	0.050	0.050

Highlighted values indicate cases for which the actual CV of class a was smaller than the specified target of 0.05.

In Table 17, the values obtained for the standard error ratios used to compare the effect of each allocation method are presented. For the Bethel/SSW ratio, there was only one case for class a where the Bethel allocation performed substantially better than SSW, while in two cases for class a SSW performed substantially better than Bethel. However, for class b, ten cases showed better standard errors when using SSW as the allocation method, and all of these cases belong to populations labeled as c. This indicates that, in this first ratio, SSW had an advantage over Bethel for the larger of the two rare classes (i.e., class b).

For the standard error ratios comparing the AvgOpt and Bethel allocation methods, AvgOpt performed substantially better in 13 cases in class a and in 8 cases in class b, while Bethel performed better than AvgOpt in only 5 cases in class b. This overall demonstrates a general superiority for AvgOpt relative to Bethel for both classes (a and b). In the comparison of AvgOpt with SSW, AvgOpt demonstrated superior performance for class a. SSW substantially outperformed AvgOpt in only four cases, all of which were in class b.

Table 17. Ratios of standard errors obtained from the different allocation methods for populations of the designed experiment.

Population	Ratio					
	Bethel/SSW		AvgOpt/Bethel		AvgOpt/SSW	
	a	b	a	b	a	b
1c	1.02	1.18	0.94	0.87	0.96	1.02
1d	1.08	1.00	0.70	1.10	0.75	1.10
2c	0.98	1.59	0.97	0.66	0.95	1.05
2d	0.92	1.04	0.99	1.00	0.91	1.04
3c	1.05	1.04	0.93	0.97	0.98	1.01
3d	1.07	1.00	0.65	1.12	0.70	1.12
4c	1.04	1.24	0.95	0.82	0.98	1.02
4d	0.88	1.08	1.00	0.98	0.88	1.06
5c	1.00	1.08	0.89	0.96	0.89	1.03
5d	1.10	1.00	0.69	1.08	0.76	1.08
6c	1.01	1.33	0.91	0.79	0.92	1.05
6d	0.97	1.01	0.96	1.01	0.92	1.03
7c	1.03	1.01	0.89	1.01	0.91	1.02
7d	1.11	1.00	0.66	1.10	0.73	1.10
8c	1.07	1.11	0.89	0.93	0.95	1.02
8d	0.96	1.05	0.97	0.98	0.93	1.03
9c	0.97	1.18	0.99	0.87	0.97	1.02
9d	1.07	1.00	0.69	1.10	0.74	1.10
10c	0.92	1.60	1.03	0.66	0.95	1.05
10d	0.97	1.00	0.90	1.05	0.87	1.05
11c	1.00	1.05	0.98	0.96	0.98	1.01
11d	1.08	1.00	0.62	1.12	0.67	1.12
12c	0.98	1.25	1.00	0.82	0.98	1.02
12d	0.91	1.02	0.90	1.06	0.82	1.08
13c	0.96	1.08	0.97	0.95	0.92	1.02
13d	1.09	1.00	0.69	1.08	0.75	1.07
14c	0.93	1.34	0.99	0.78	0.92	1.05
14d	1.01	1.00	0.86	1.04	0.87	1.04
15c	0.98	1.02	0.96	1.00	0.94	1.01
15d	1.10	1.00	0.64	1.09	0.70	1.09
16c	0.99	1.12	0.97	0.91	0.96	1.02
16d	0.95	1.01	0.89	1.05	0.84	1.06

3.4. Correlation analysis

Table 18 shows the results obtained from the correlation analysis for the real populations evaluated in this study. The table shows correlations separately for each of the two target classes a and b. The correlations examine the association between the standard error ratios of the different allocation methods with several features of the populations: user's accuracy, producer's accuracy, overall accuracy, % area, "a-b" (which is the difference between the % area of the two target classes), and "SD ratio" which is the ratio of the standard deviations (square root of equation 2) of the two target classes. Each cell contains a correlation coefficient that measures the relationship between the standard error ratios and the population feature. Cells highlighted in yellow indicate the stronger correlations (positive or negative). The orange background indicates the highest values obtained in the analysis. The correlation values ranged from -0.74 to 0.82, suggesting moderate inverse (negative) correlations to strong direct (positive) correlations.

For the Bethel/SSW ratio in class a, the correlations were generally low, suggesting little or no relationship with population features. However, the correlation of Bethel/SSW with “a-b” was 0.82 indicating that as the difference between the areas of classes a and b increased, the Bethel/SSW standard error ratio increased (i.e., a larger difference resulting in a larger ratio would favor SSW). For class b, the strongest correlations were a positive correlation of 0.40 with producer's accuracy and a negative correlation of -0.37 with overall accuracy. Similarly the correlations of the standard error ratio of AvgOpt/Bethel did not have strong correlations with the population features as the two largest correlations both occurred with producer’s accuracy (classes a and b) but were only -0.34 and -0.44. The AvgOpt/SSW ratios had strong correlations with the "a-b" feature for both classes, this being positive for class a (0.81) and negative for class b (-0.74). These correlations suggest that the magnitude of the difference in percent area of the two rare target classes was strongly associated with the AvgOpt/SSW standard error ratios. There was also a strong positive correlation of 0.67 with % area for class b which would indicate that as the percent area of class b increased, the AvgOpt/SSW ratio increased (larger ratios would favor precision of SSW). Finally, for the SD Ratio feature, a notable correlation for both classes were observed, this being negative for class a (-0.62) and positive for class b (0.64).

Table 18. Correlations between population characteristics and standard error ratios for real populations.

SE Ratio	Correlation - Real Populations											
	SE Ratio vs user's		SE Ratio vs producer's		SE Ratio vs overall		SE Ratio vs % Area		SE Ratio vs a-b		SE Ratio vs SD Ratio	
	a	b	a	b	a	b	a	b	a	b	a	b
Bethel /SSW	0.02	0.15	0.06	0.40	0.23	-0.37	-0.22	0.20	0.82	-0.30	-0.30	0.17
AvgOpt /Bethel	-0.11	-0.16	-0.34	-0.44	-0.14	0.32	0.19	0.01	-0.23	0.08	-0.20	-0.05
AvgOpt /SSW	-0.09	0.16	-0.24	-0.18	0.13	0.07	-0.04	0.67	0.81	-0.74	-0.62	0.64

Table 19 shows the results obtained from the correlation analysis for the populations of the designed experiment. One difference in the correlation analysis of these populations versus the real populations was that the correlations of the standard error ratios with the population features % area, a-b (difference in area), and SD Ratio were not calculated because the area proportions of the two target classes were the same for all populations (class a has 0.01 and class b has 0.05).

Correlation values range from -0.46 to 0.42, indicating moderate inverse correlations to moderate direct correlations. The correlations for these populations were smaller in magnitude than those for the real populations. For class a, the Bethel/SSW ratio was not strongly correlated with any of the population features, while for class b there was a moderate positive correlation with producer's accuracy (0.42). Similarly for class a and the AvgOpt/Bethel standard error ratios, the correlations with the accuracy features were small (0.10 was the largest). For class b, the AvgOpt/Bethel ratio had a correlation of -0.46 with producer's accuracy of class b. This would indicate that as the producer's accuracy of class b increased, the AvgOpt/Bethel ratio would decrease (i.e., more advantage to AvgOpt). Finally, the correlations between the AvgOpt/SSW ratios and accuracy features were small for both target classes with only the correlation in class b (-0.25) of AvgOpt/SSW with producer's accuracy having a magnitude greater than 0.10 in absolute value.

Table 19. Correlations for constructed populations.

SE Ratio	Correlation - Experimental Design					
	SE Ratio vs user's		SE Ratio vs producer's		SE Ratio vs overall	
	a	b	a	b	a	b
Bethel /SSW	-0.17	-0.28	0.14	0.42	-0.12	-0.16
AvgOpt /Bethel	0.07	0.23	-0.10	-0.46	0.09	0.10
AvgOpt /SSW	0.00	-0.01	-0.05	-0.25	0.06	-0.04

3.5. Regression analysis

For this section it is important to recognize that a high R^2 value does not necessarily imply that the model is suitable for making predictions or that the variables included are the most relevant; it simply indicates the proportion of the variability explained by the model. The purpose here is to examine the association of the standard error ratios with the full suite of population features together by using R^2. Table 20 shows the results obtained from the multiple linear regression analysis for the real populations. The population features are the same ones used in the correlations shown in Table 18. The table shows the R^2 values obtained from a regression analysis for three ratios: Bethel/SSW, AvgOpt/Bethel, and AvgOpt/SSW. These values are calculated for the two target classes a and b. Yellow cells highlight R^2 values, possibly indicating greater relevance in the analysis. The R^2 values range from 0.197 to 0.801, indicating the proportion of the variance in the dependent variable that is explained by the regression model.

The Bethel/SSW standard error ratios for class a had an R^2= 0.705 with the features of the real populations indicating that 70.5% of the variability in the standard error ratios was explained by the population features. For class b R^2 = 0.510, which indicates that 51% of the variability was explained by the model. The R^2 values were smaller for the AvgOpt/Bethel ratios as class a had R^2= 0.197 and class b had R^2= 0.443. For the AvgOpt/SSW ratios and class a, R^2= 0.801 was the strongest association observed. For class b we found R^2= 0.643. For the real populations, the proportions of the two target classes appear to have a strong association with the standard error ratios.

Table 20. R^2 values for real populations.

Ratio	**Real Populations**	
	R^2	
	a	**b**
Bethel/SSW	0.705	0.510
AvgOpt/Bethel	0.197	0.443
AvgOpt/SSW	0.801	0.643

Table 21 shows the results obtained from the regression analysis for experimental design populations. These R^2 values were smaller than those observed for the real populations, perhaps reflecting the absence of the important influence of the target class proportions observed in the real populations. The largest R^2 for the designed experiment was about 0.30 indicating that the accuracy features (user's, producer's and overall) did not explain a large percent of the variability in the standard error ratios.

In the Bethel/SSW ratio we have that the R^2 of class a indicates a low fit of the model, while in class b we have that the R^2 shows a moderate fit. For the AvgOpt/Bethel ratio, only class b shows a moderate fit. Finally, for the AvgOpt/SSW ratio, both classes do not show a substantial association between the standard error ratio and population features.

Table 21. R^2 values for constructed populations.

Ratio	Experimental Design	
	R^2	
	a	b
Bethel/SSW	0.065	0.294
AvgOpt/Bethel	0.025	0.301
AvgOpt/SSW	0.007	0.094

4. DISCUSSION

The Average Optimal method proved to be an effective and easy to calculate option. Although the SSW method had some advantage over Average Optimal in a few cases, usually for the more common class b, the Average Optimal method performed as well or better than the Bethel and SSW methods in most cases. This suggests that in practical applications, the Average Optimal method would be a viable choice because of its simplicity and effectiveness.

Cochran (1977, p.103 and Sec. 5A.1) noted that "the optima in the allocation problem are rather flat." The stratum specific population P_h values (i.e., proportion of area of the target class in stratum h) are not known so a best guess of these values must be used in the optimal allocation calculations. The flat optimum provides assurance that if the P_h values are close to the true values, the sample allocation will not be too far from the optimum. The existence of an "optimal plane" has important implications for the practice of sample allocation. It suggests that researchers can have some flexibility in sample allocation without substantially compromising the precision of their estimates. This is particularly useful in situations where available data or logistical constraints limit the ability to achieve an exact optimal allocation. Furthermore, this flexibility can facilitate the implementation of simpler and less costly allocation strategies, without sacrificing the quality of the results.

This study used a single coefficient of variation (CV) value for both target classes in the Bethel method (i.e., equal CV for all target estimates) and the same importance weight for both classes in the SSW method. Furthermore, in the experimental design, the same areas in classes a and b were maintained in all populations with the different combinations of two levels of user's and producer's accuracies. This uniformity of conditions could represent a limitation in terms of generality of results. To obtain a more complete view of the performance of the Bethel and SSW methods, it would be valuable to explore different CV values and importance weights, as well as varying the areas in classes a and b of the designed experiment. This would allow evaluating how a broader range of factors influence the effectiveness of the allocation methods. Likewise, including additional real populations and a broader set of populations in the designed experiment would provide a broader basis for practical recommendations, allowing allocation strategies to be adapted to specific and diversified contexts. Another extension of this study would be to evaluate these allocation methods for 3 or 4 target classes instead of just 2 target classes which was the focus of the results presented. A few examples with more than 2 target classes are provided in the appendix to show some preliminary results. Three of the examples have 3 target classes and one example has 4 target classes.

5. CONCLUSIONS

This research investigated methods for optimally allocating a stratified random sample when estimating areas of two or more rare classes. In practice, users can choose an allocation method based on their preference for how the optimization is structured (e.g., Bethel focuses on achieving a specified CV whereas SSW minimizes the sum of the variance of the target estimates while allowing for specification of importance weights). Average Optimal is simple to compute as it requires applying the Neyman optimal allocation to each target estimate individually and then averaging those values over the set of target estimates. Based on the results of this study, the Average Optimal method generally demonstrated equal or better performance than the Bethel and SSW methods. Although this method lacks formal mathematical justification as the optimal approach (i.e., it assumes that "splitting the difference" between separate Neyman optimal allocations may offer a simple and effective solution for estimating two or more targets), it often yielded smaller standard errors than the Bethel and SSW allocations. For the comparison of Bethel and SSW, the SSW method was generally better and yielded smaller standard errors for the class with the larger percent area (in the two target class applications). Bethel allocation tended to be closer to Neyman allocation of the rarer class and therefore could yield smaller standard errors for the rarer class relative to the SSW allocation. Further analysis is recommended to fully evaluate these methods, particularly by exploring different approaches for the Bethel and SSW methods. Investigating how results are influenced by varying coefficients of variation (CV) in the Bethel method and different levels of importance weights in the SSW method could provide additional insights.

6. LITERATURE CITED

Aryal, R. R., Wespestad, C., Kennedy, R., Dilger, J., Dyson, K., Bullock, E., Khanal, N., Kono, M., Poortinga, A., Saah, D., & Tenneson, K. (2021). Lessons learned while implementing a time-series approach to forest canopy disturbance detection in Nepal. *Remote Sensing*, *13*(14), 2666. https://doi.org/10.3390/rs13142666

Badjana, H., Olofsson, P., Woodcock, C., Helmschrot, J., Wala, K., & Koffi, A. (2017). Mapping and estimating land change between 2001 and 2013 in a heterogeneous landscape in West Africa: Loss of forestlands and capacity building opportunities. *International Journal of Applied Earth Observation and Geoinformation*, *63*, 15–23. https://doi.org/10.1016/j.jag.2017.07.006

Bethel, J. (1989). Sample allocation in multivariate surveys. *Survey Methodology*, 15(1), 47–57.

Chen, S., Olofsson, P., Saphangthong, T., & Woodcock, C. E. (2023). Monitoring shifting cultivation in Laos with Landsat time series. *Remote Sensing of Environment*, 288, 113507. https://doi.org/10.1016/j.rse.2023.113507

Chen, S., Woodcock, C. E., Bullock, E. L., Arévalo, P., Torchinava, P., Peng, S., & Olofsson, P. (2021). Monitoring temperate forest degradation on Google Earth Engine using Landsat time series analysis. *Remote Sensing of Environment*, 265, 112648. https://doi.org/10.1016/j.rse.2021.112648

Cochran, W. G. (1977). Sampling Techniques (3rd edition). John Wiley & Sons.

De Meo. (2022, October 12). Package 'bethel.' https://cran.r-project.org/web/packages/bethel/bethel.pdf

Department of Forestry Lao PDR. (2018). *Activity Data Report for the ER Program of Lao PDR* (p. 32). https://www.forestcarbonpartnership.org/system/files/documents/Annex%2010%20-

%20LaoPDR_ERPD%20AD%20%20Report_0323.pdf

Dymond, J. R., Shepherd, J. D., Newsome, P. F., Gapare, N., Burgess, D. W., & Watt, P. (2012). Remote sensing of land-use change for Kyoto Protocol reporting: The New Zealand case. *Environmental Science & Policy*, *16*, 1–8. https://doi.org/10.1016/j.envsci.2011.11.011

Goetz, S. J., Hansen, M., Houghton, R. A., Walker, W., Laporte, N., & Busch, J. (2015). Measurement and monitoring needs, capabilities and potential for addressing reduced emissions from deforestation and forest degradation under REDD+. *Environmental Research Letters*, 10(12), 123001. https://doi.org/10.1088/1748-9326/10/12/123001

Ministry of Agriculture and Forests Forest National Corporation REDD+ Programme. (2020). *Republic of Sudan's Forest Reference Level (FRL) Submission to the UNFCCC* (p. 73). https://redd.unfccc.int/media/sudans_modified_frl_submission_webposting.pdf

Ministry of Environment, Forest and Climate Change (MoEFCC) Government of the People's Republic of Bangladesh. (2018). *The submission of Bangladesh's Forest Reference Level for REDD+ under the UNFCCC* (p. 155). https://redd.unfccc.int/media/frl-report_revised_17_july2019_f.pdf

Ministry of Natural Resources and Environment. (2020). *Forest Reference Emission Level and Forest Reference Level Thailand* (p. 137). https://redd.unfccc.int/media/thailand_frel_frl_report.pdf

Neyman, J. (1934). On the Two Different Aspects of the Representative Method: The method of stratified sampling and the method of purposive selection. *Journal of the Royal Statistical Society*, 97(4), 558. https://doi.org/10.2307/2342192

Olofsson, P., Arévalo, P., Espejo, A. B., Green, C., Lindquist, E., McRoberts, R. E., & Sanz, M. J. (2020). Mitigating the effects of omission errors on area and area change estimates. *Remote Sensing of Environment*, *236*, 111492. https://doi.org/10.1016/j.rse.2019.111492

Olofsson, P., Foody, G., Stehman, S., & Woodcock, C. (2013). Making better use of accuracy data in land change studies: Estimating accuracy and area and quantifying uncertainty using stratified estimation. *Remote Sensing of Environment. 129.* 122–131. https://doi.org/10.1016/j.rse.2012.10.031

Olofsson, P., Foody, G. M., Herold, M., Stehman, S. V., Woodcock, C. E., & Wulder, M. A. (2014). Good practices for estimating area and assessing accuracy of land change. *Remote Sensing of Environment*, *148*, 42–57. https://doi.org/10.1016/j.rse.2014.02.015

Särndal, C. E., & Lundström, S. (2005). *Estimation in Surveys with Nonresponse*. John Wiley & Sons.

Särndal, C. E., Swensson, B., & Wretman, J. (1992). *Model Assisted Survey Sampling*. Springer New York.

Stehman, S. V. (2013). Estimating area from an accuracy assessment error matrix. *Remote Sensing of Environment*, 132, 202–211. https://doi.org/10.1016/j.rse.2013.01.016

Stehman, S. V., & Wagner, J. E. (2024). Choosing a sample size allocation to strata based on trade-offs in precision when estimating accuracy and area of a rare class from a stratified sample. *Remote Sensing of Environment*, *300*, 113881. https://doi.org/10.1016/j.rse.2023.113881

Wagner, J. E., & Stehman, S. V. (2015). Optimizing sample size allocation to strata for estimating area and map accuracy. *Remote Sensing of Environment*, *168*, 126–133. https://doi.org/10.1016/j.rse.2015.06.027

Wickham, J., Stehman, S. V., Sorenson, D. G., Gass, L., & Dewitz, J. A. (2023). Thematic accuracy assessment of the NLCD 2019 land cover for the conterminous United

States. *GIScience & Remote Sensing*, *60*(1), 2181143. https://doi.org/10.1080/15481603.2023.2181143

Yang, X., Zhu, Z., Qiu, S., Kroeger, K. D., Zhu, Z., & Covington, S. (2022). Detection and characterization of coastal tidal wetland change in the northeastern US using Landsat time series. *Remote Sensing of Environment*, *276*, 113047. https://doi.org/10.1016/j.rse.2022.113047

7. APPENDICES

7.1. Results for the more than 2 target classes

Appendix 1 shows that Average Optimal and SSW methods generally achieve lower standard errors, suggesting more precise estimates, compared with the Proportional method, while Bethel does not present any important difference. Differences in sample sizes across strata suggest that each method has its specific strengths depending on the strata characteristics.

Appendix 1. Sample allocations and Area Percentages for Three Target Classes

Stratum	**P_h**			**Optimal**				**Proportional**	**Equal**	**Bethel**	**SSW**	
	Barren Land	**Water**	**Wetland**	**30**	**10**	**90**	**Average**					
1	0.0021	0.9604	0.0163	197	1119	207	508	177	1250	209	319	
2	0.0014	0.0017	0.0035	542	795	328	555	602	1250	561	369	
3	0.6402	0.0264	0.0128	1182	525	105	604	101	1250	245	398	
4	0.0026	0.0004	0.0090	3139	1624	2194	2319	2510	1250	3251	2089	
5	0.0036	0.0004	0.0004	3305	1544	439	1763	2268	1250	3394	1150	
7	0.0001	0.0007	0.0193	272	1205	1755	1078	1384	1250	452	1475	
8	0.0004	0.0004	0.0100	1180	1568	2149	1633	2342	1250	1290	1844	
9	0.0001	0.0065	0.5943	183	1619	2824	1542	623	1250	598	2357	
			Total	*10000*	*10000*	*10000*	*10000*	*10006*	*10000*	*10000*	*10000*	**SRS**
			SE (30)	**0.041**	**0.054**	**0.087**	**0.049**	**0.064**	**0.057**	**0.048**	**0.057**	**0.090**
			SE (10)	**0.055**	**0.031**	**0.044**	**0.033**	**0.043**	**0.032**	**0.043**	**0.035**	**0.134**
			SE (90)	**0.268**	**0.127**	**0.108**	**0.125**	**0.152**	**0.140**	**0.174**	**0.113**	**0.207**

Wickham et al. (2023) case. Barren Land (area of 0.82%), Water (1.82%), and Wetland (4.50%).

Appendix 2 shows that the average optimal method performed better for all three classes compared to proportional allocation, while SSW and Bethel do not represent a big advantage.

Appendix 2. Sample allocations and Area Percentages for Three Target Classes

Stratum	**P_h**			**Optimal**				**Proportional**	**Equal**	**Bethel**	**SSW**	
	Forest Loss	**Reforestation**	**Savannah Loss**	**Forest Loss**	**Reforestation**	**Savannah Loss**	**Average**					
1	0.0252	0.0005	0.0005	4872	475	281	1876	1984	1250	1792	1438	
2	0.0003	0.0077	0.0335	976	3625	4412	3005	3882	1250	3230	3531	
3	0.0037	0.0146	0.0146	2585	3499	2073	2719	2732	1250	3233	2232	
4	0.0087	0.0087	0.0087	167	114	67	116	115	1250	118	84	
5	0.0588	0.0588	0.0588	63	43	25	44	17	1250	23	31	
6	0.4819	0.0120	0.1205	650	97	171	306	83	1250	113	228	
7	0.0010	0.0192	0.7383	506	1525	2897	1643	1043	1250	1363	2181	
8	0.0075	0.7463	0.0075	181	622	73	292	134	1250	127	274	
Total				*10000*	*10000*	*10000*	*10000*	*9990*	*10000*	*10000*	*10000*	**SRS**
SE(Forest Loss)				**0.064**	**0.153**	**0.193**	**0.084**	**0.092**	**0.102**	**0.091**	**0.095**	**0.102**
SE(Reforestation)				**0.149**	**0.094**	**0.119**	**0.102**	**0.109**	**0.141**	**0.107**	**0.103**	**0.138**
SE(Savannah Loss)				**0.310**	**0.178**	**0.158**	**0.183**	**0.195**	**0.254**	**0.187**	**0.170**	**0.294**

Badjana et al. (2017) RF Classification case. Forest Loss has a reference area of 1.05%, Reforestation has area of 1.94%, and Savannah Loss has area of 9.54%.

In Appendix 3 it is observed again that both the Average Optimal method and the SSW have a better performance in the first two classes evaluated, but does not represent an advantage in the third class, while Bethel only shows a slight advantage in all classes.

Appendix 3. Sample allocations and Area Percentages for Three Target Classes

Stratum	P_h			Optimal				Proportional	Equal	Bethel	SSW	
	Forest Loss	**Reforestation**	**Savannah Loss**	**Forest Loss**	**Reforestation**	**Savannah Loss**	**Average**					
1	0.0065	0.0261	0.0007	2023	2040	185	1416	1533	1250	2106	1011	
2	0.0002	0.0131	0.0744	1108	4343	5658	3703	4571	1250	4482	4786	
3	0.0004	0.0044	0.0131	784	1261	1229	1091	2292	1250	1303	1115	
4	0.0044	0.0044	0.0044	245	125	71	147	225	1250	131	95	
5	0.0278	0.0278	0.0278	97	49	28	58	36	1250	52	37	
6	0.3321	0.0185	0.2583	4184	609	1119	1970	542	1250	629	1302	
7	0.0142	0.0285	0.4843	1363	975	1655	1331	702	1250	1008	1386	
8	0.0069	0.5556	0.0069	196	597	56	283	144	1250	289	269	
			Total	*10000*	*10000*	*10000*	*10000*	*10045*	*10000*	*10000*	*10000*	**SRS**
			SE (Forest Loss)	**0.061**	**0.113**	**0.125**	**0.074**	**0.121**	**0.087**	**0.111**	**0.088**	**0.142**
			SE (Reforestation)	**0.185**	**0.120**	**0.224**	**0.130**	**0.132**	**0.174**	**0.122**	**0.131**	**0.147**
			SE (Savannah Loss)	**0.386**	**0.246**	**0.212**	**0.240**	**0.251**	**0.368**	**0.243**	**0.223**	**0.279**

Badjana et al. (2017) SVM Classification case. Forest Loss (2.05%), Reforestation (2.22%), and Savannah Loss (8.54%).

In Appendix 4, the Average Optimal and SSW methods demonstrate significantly better performance compared to the proportional method. In contrast, the Bethel method, while showing some advantages, does not perform as well as the other methods analyzed.

Appendix 4. Sample allocations and Area Percentages for Three Target Classes

Stratum	P_h			Optimal				Proportional	Equal	Bethel	SSW	
	Degradation High	Degradation Low	Deforestation	Degradation High	Degradation Low	Deforestation	Average					
1	0.0533	0.0400	0.4133	2052	711	3557	2107	211	1250	349	1602	
2	0.2494	0.2494	0.0324	577	229	187	331	31	1250	51	259	
3	0.1049	0.3277	0.0084	1575	958	371	968	119	1250	197	890	
4	0.1607	0.1607	0.1607	99	39	78	72	6	1250	10	52	
5	0.0133	0.0533	0.0133	52	41	41	45	11	1250	18	38	
6	0.0019	0.0019	0.0132	999	397	2091	1162	536	1250	464	913	
7	0.0001	0.0023	0.0001	4081	7402	3228	4904	8932	1250	8649	5947	
8	0.0058	0.0058	0.0058	565	224	447	412	172	1250	263	298	
			Total	*10000*	*10000*	*10000*	*10000*	*10018*	*10000*	*10000*	*10000*	**SRS**
			SE (Degradation High)	**0.023**	**0.030**	**0.029**	**0.024**	**0.057**	**0.033**	**0.045**	**0.025**	**0.091**
			SE (Degradation Low)	**0.071**	**0.058**	**0.083**	**0.066**	**0.081**	**0.124**	**0.070**	**0.062**	**0.089**
			SE (Deforestation)	**0.034**	**0.052**	**0.029**	**0.033**	**0.080**	**0.044**	**0.065**	**0.037**	**0.099**

Submission of Bangladesh's Forest Reference Level for REDD+ under the UNFCCC (2018) case. Degradation High (0.36%), Degradation Low (0.80%), and Deforestation (1.00%).

As shown in Appendix 5, it is suggested that Bethel and Sarndal do not exhibit any advantage as allocation methods when dealing with three target classes, compared to Average Optimal, although SSW performs better than Bethel.

Appendix 5. Ratios of standard errors obtained from the different allocation methods for real populations with 3 target classes.

Source	**% Area**			**Ratio**								
				Bethel/SSW			**AvgOpt/Bethel**			**AvgOpt/SSW**		
	a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
Wickham et al. (2023)	0.82	1.82	4.50	0.85	1.21	1.54	1.01	0.76	0.72	0.86	0.92	1.11
Badjana et al. (2017) RF Classification	1.05	1.94	9.54	0.95	1.04	1.10	0.93	0.96	0.98	0.88	0.99	1.08
Badjana et al. (2017) SVM Classification	2.05	2.22	8.54	1.27	0.93	1.09	0.67	1.07	0.99	0.85	0.99	1.07
Bangladesh's Forest Reference Level for REDD+ under the UNFCCC (2018)	0.36	0.80	1.00	1.78	1.13	1.79	0.53	0.95	0.51	0.95	1.07	0.91

In the following case, four target classes were analyzed (Appendix 6). Average and SSW demonstrate better performance compared to the proportional method, and the standard errors produced by both methods are very similar. It could be suggested that SSW may yield better performance and results when dealing with rarer classes, although further analysis and testing of different weights for each class in the method would be needed.

Appendix 6. Sample allocations and Area Percentages for Four Target Classes

Stratum	P_h				Optimal					Proportional	Equal	Bethel	SSW	
	Restoration	Degradation	Reforestation	Deforestation	Restoration	Degradation	Reforestation	Deforestation	Average					
1	0.0030	0.0327	0.0030	0.8631	323	616	211	2290	860	336	1667	359	838	
2	0.0333	0.6000	0.0333	0.0667	380	606	248	594	457	120	1667	184	459	
3	0.0315	0.0045	0.7252	0.0045	684	153	1143	295	569	222	1667	340	685	
4	0.6441	0.0169	0.1017	0.0339	498	79	206	212	249	59	1667	90	227	
5	0.0026	0.0089	0.0064	0.0013	7635	8266	7878	6072	7463	8535	1667	8494	7448	
6	0.0014	0.0014	0.0014	0.0014	479	280	313	538	403	738	1667	534	343	
Total					*10000*	*10000*	*10000*	*10000*	*10000*	*10010*	*10000*	*10000*	*10000*	**SRS**
SE(Restoration)					**0.057**	**0.068**	**0.059**	**0.065**	**0.058**	**0.070**	**0.107**	**0.063**	**0.059**	**0.085**
SE (Degradation)					**0.103**	**0.097**	**0.107**	**0.108**	**0.100**	**0.109**	**0.198**	**0.104**	**0.100**	**0.126**
SE (Reforestation)					**0.089**	**0.114**	**0.087**	**0.107**	**0.092**	**0.105**	**0.169**	**0.096**	**0.091**	**0.149**
SE (Deforestation)					**0.076**	**0.063**	**0.091**	**0.050**	**0.057**	**0.079**	**0.081**	**0.075**	**0.058**	**0.174**

Activity Data Report for the ER Program of Lao PDR (2018) case. Restoration has an area of 0.73%, Degradation has area of 1.62%, Reforestation has area 2.28%, and Deforestation has area 3.13%.

In Appendix 7, we observe that no notable advantages are shown between the methods. However, it is recommended to conduct more population studies and consider different characteristics of each method, such as varying CVs for the Bethel method.

Appendix 7. Ratios of standard errors obtained from the different allocation methods for real populations with 4 target classes.

Source	% Area				Ratio											
					Bethel/SSW				AvgOpt/Bethel				AvgOpt/SSW			
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d
LAO	0.73	1.62	2.28	3.13	1.08	1.03	1.06	1.30	0.92	0.97	0.96	0.76	1.00	1.00	1.01	0.99

7.2. Sample allocations for Real Populations R1-R17 (see Table 9 for details of each population)

Appendix 8. Population R2.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Forest Loss	Savannah Loss	Forest Loss	Savannah Loss	Average					
1	0.0065	0.0007	2023	185	1104	1533	1250	1039	551	
2	0.0002	0.0744	1108	5658	3383	4571	1250	4060	5109	
3	0.0004	0.0131	784	1229	1007	2292	1250	962	1127	
4	0.0044	0.0044	245	71	158	225	1250	135	90	
5	0.0278	0.0278	97	28	62	36	1250	54	36	
6	0.3321	0.2583	4184	1119	2651	542	1250	2275	1482	
7	0.0142	0.4843	1363	1655	1509	702	1250	1367	1534	
8	0.0069	0.0069	196	56	126	144	1250	108	72	
Total			*10000*	*10000*	*10000*	*10045*	*10000*	*10000*	*10000*	**SRS**
SE (Forest Loss)			**0.061**	**0.125**	**0.071**	**0.121**	**0.087**	**0.075**	**0.092**	**0.142**
SE (Savannah Loss)			**0.386**	**0.212**	**0.245**	**0.251**	**0.368**	**0.233**	**0.216**	**0.279**

Appendix 9. Population R3.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Barren Land	Wetland	30	90	Average					
1	0.0021	0.0163	197	207	202	177	1250	209	188	
2	0.0014	0.0035	542	328	435	602	1250	561	332	
3	0.6402	0.0128	1182	105	643	101	1250	245	394	
4	0.0026	0.0090	3139	2194	2666	2510	1250	3251	2139	
5	0.0036	0.0004	3305	439	1872	2268	1250	3394	1136	
7	0.0001	0.0193	272	1755	1014	1384	1250	452	1507	
8	0.0004	0.0100	1180	2149	1665	2342	1250	1290	1882	
9	0.0001	0.5943	183	2824	1503	623	1250	598	2422	
		Total	*10000*	*10000*	*10000*	*10006*	*10000*	*10000*	*10000*	**SRS**
		SE (30)	**0.041**	**0.087**	**0.048**	**0.064**	**0.057**	**0.048**	**0.058**	**0.090**
		SE (90)	**0.268**	**0.108**	**0.126**	**0.152**	**0.140**	**0.174**	**0.113**	**0.207**

Appendix 10. Population R4.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	New Plantation	Deforestation	New Plantation	Deforestation	Average					
1	0.0002	0.0021	1403	1457	1430	4771	2000	1428	1441	
2	0.0100	0.0500	2204	1578	1891	1090	2000	2241	1636	
3	0.0061	0.0517	5686	5294	5490	3600	2000	5782	5292	
4	0.0020	0.4668	450	1627	1039	491	2000	458	1541	
5	0.9091	0.0227	257	44	150	44	2000	92	89	
		Total	*10000*	*10000*	*10000*	*9996*	*10000*	*10000*	*10000*	**SRS**
		SE (New Plantation)	**0.049**	**0.054**	**0.050**	**0.062**	**0.069**	**0.050**	**0.052**	**0.086**
		SE (Deforestation)	**0.175**	**0.151**	**0.154**	**0.190**	**0.200**	**0.173**	**0.151**	**0.214**

Appendix 11. Population R6.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Deg_re	Deg_no_re	Deg_re	Deg_no_re	Average					
1	0.0303	0.0303	104	72	88	33	1429	81	82	
2	0.1099	0.1099	522	364	443	91	1429	405	413	
3	0.3688	0.3254	4076	2764	3420	461	1429	3088	3185	
4	0.0076	0.6818	210	787	498	132	1429	681	642	
5	0.0034	0.0687	312	942	627	291	1429	816	776	
6	0.0029	0.0086	3419	4123	3771	3491	1429	3884	3825	
7	0.0002	0.0002	1358	948	1153	5493	1429	1045	1077	
Total			*10000*	*10000*	*10000*	*9992*	*10000*	*10000*	*10000*	**SRS**
SE (Degradation w/Regeneration)			**0.055**	**0.059**	**0.056**	**0.114**	**0.080**	**0.058**	**0.057**	**0.059**
SE (Degradation no Regeneration)			**0.091**	**0.078**	**0.080**	**0.138**	**0.108**	**0.079**	**0.079**	**0.171**

Appendix 12. Population R7.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Gain	Loss	Gain	Loss	Average					
1	0.0977	0.0703	522	348	435	256	2500	450	420	
2	0.0311	0.1886	651	1135	893	546	2500	657	978	
3	0.0398	0.0550	3642	3284	3463	2711	2500	3670	3408	
4	0.0137	0.0236	5184	5233	5208	6488	2500	5222	5193	
Total			*10000*	*10000*	*10000*	*10001*	*10000*	*10000*	*10000*	**SRS**
SE (Gain)			**0.146**	**0.148**	**0.146**	**0.152**	**0.186**	**0.146**	**0.147**	**0.153**
SE (Loss)			**0.192**	**0.188**	**0.189**	**0.198**	**0.237**	**0.192**	**0.189**	**0.201**

Appendix 13. Population R8.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Degradation	**Deforestation**	**Degradation**	**Deforestation**	**Average**					
1	0.0327	0.8631	616	2290	1453	336	1667	611	1154	
2	0.6000	0.0667	606	594	600	120	1667	218	585	
3	0.0045	0.0045	153	295	224	222	1667	160	187	
4	0.0169	0.0339	79	212	145	59	1667	84	116	
5	0.0089	0.0013	8266	6072	7169	8535	1667	8633	7617	
6	0.0014	0.0014	280	538	409	738	1667	293	341	
		Total	*10000*	*10000*	*10000*	*10010*	*10000*	*10000*	*10000*	**SRS**
		SE (Degradation)	**0.097**	**0.108**	**0.101**	**0.109**	**0.198**	**0.100**	**0.099**	**0.126**
		SE (Deforestation)	**0.063**	**0.050**	**0.052**	**0.079**	**0.081**	**0.065**	**0.055**	**0.174**

Appendix 14. Population R9.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Water	**Wetland**	**10**	**90**	**Average**					
1	0.9604	0.0163	1119	207	663	177	1250	292	356	
2	0.0017	0.0035	795	328	562	602	1250	355	374	
3	0.0264	0.0128	525	105	315	101	1250	143	171	
4	0.0004	0.0090	1624	2194	1909	2510	1250	2141	2102	
5	0.0004	0.0004	1544	439	991	2268	1250	516	583	
7	0.0007	0.0193	1205	1755	1480	1384	1250	1711	1676	
8	0.0004	0.0100	1568	2149	1859	2342	1250	2097	2057	
9	0.0065	0.5943	1619	2824	2222	623	1250	2744	2682	
		Total	*10000*	*10000*	*10000*	*10006*	*10000*	*10000*	*10000*	**SRS**
		SE (10)	**0.031**	**0.044**	**0.033**	**0.043**	**0.032**	**0.040**	**0.038**	**0.134**
		SE (90)	**0.127**	**0.108**	**0.114**	**0.152**	**0.140**	**0.109**	**0.109**	**0.207**

Appendix 15. Population R10.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Deforestation	**Afforestation**	**Deforestation**	**Afforestation**	**Average**					
1	0.0003	0.0003	2044	3449	2746	3559	2500	3582	2426	
2	0.9383	0.0014	54	385	220	28	2500	50	194	
3	0.0011	0.9711	377	407	392	216	2500	387	1058	
4	0.0003	0.0010	7525	5759	6642	6201	2500	5981	6322	
		Total	*10000*	*10000*	*10000*	*10003*	*10000*	*10000*	*10000*	**SRS**
		SE (Deforestation)	**0.017**	**0.021**	**0.018**	**0.021**	**0.023**	**0.019**	**0.018**	**0.054**
		SE (Afforestation)	**0.033**	**0.029**	**0.032**	**0.037**	**0.042**	**0.033**	**0.030**	**0.146**

Appendix 16. Population R11.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Deforestation	Degradation	Deforestation	Degradation	Average					
1	0.6250	0.0313	657	66	361	32	1667	69	181	
2	0.0110	0.2198	402	446	424	91	1667	196	426	
3	0.0016	0.6874	1048	3349	2199	611	1667	1313	3120	
4	0.0345	0.0690	2244	869	1557	290	1667	623	994	
5	0.0003	0.0115	2506	4394	3450	3491	1667	3460	4129	
6	0.0002	0.0002	3143	876	2010	5492	1667	4339	1150	
Total			*10000*	*10000*	*10000*	*10007*	*10000*	*10000*	*10000*	**SRS**
SE (Deforestation)			**0.024**	**0.038**	**0.026**	**0.046**	**0.028**	**0.034**	**0.032**	**0.058**
SE (Degradation)			**0.118**	**0.085**	**0.093**	**0.144**	**0.118**	**0.109**	**0.085**	**0.218**

Appendix 17. Population R12.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Forest Gain	Deforestation	Forest Gain	Deforestation	Average					
1	0.0050	0.6965	281	1071	676	201	2500	504	892	
2	0.5960	0.0066	1472	142	807	151	2500	379	717	
3	0.0003	0.0062	1124	2922	2023	3201	2500	2550	2468	
4	0.0031	0.0062	7124	5866	6495	6450	2500	6567	5923	
Total			*10000*	*10000*	*10000*	*10003*	*10000*	*10000*	*10000*	**SRS**
SE (Forest Gain)			**0.050**	**0.079**	**0.053**	**0.077**	**0.074**	**0.060**	**0.056**	**0.105**
SE (Deforestation)			**0.111**	**0.086**	**0.092**	**0.102**	**0.115**	**0.090**	**0.089**	**0.140**

Appendix 18. Population R13.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Deforestation	Deg_no_re	Deforestation	Deg_no_re	Average					
1	0.6061	0.0303	659	72	366	33	1429	73	201	
2	0.0110	0.1099	388	364	376	91	1429	202	353	
3	0.0022	0.3254	876	2764	1820	461	1429	1021	2554	
4	0.0076	0.6818	468	787	627	132	1429	292	736	
5	0.0344	0.0687	2166	942	1554	291	1429	645	1068	
6	0.0003	0.0086	2414	4123	3269	3491	1429	3446	3855	
7	0.0002	0.0002	3029	948	1988	5493	1429	4321	1233	
Total			*10000*	*10000*	*10000*	*9992*	*10000*	*10000*	*10000*	**SRS**
SE (Deforestation)			**0.024**	**0.037**	**0.027**	**0.048**	**0.03**	**0.034**	**0.031**	**0.059**
SE (Degradation no Regeneration)			**0.105**	**0.078**	**0.085**	**0.138**	**0.108**	**0.101**	**0.079**	**0.171**

Appendix 19. Population R14.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Barren Land	Water	30	10	Average					
1	0.0021	0.9604	197	1119	658	177	1250	219	634	
2	0.0014	0.0017	542	795	668	602	1250	601	592	
3	0.6402	0.0264	1182	525	854	101	1250	254	914	
4	0.0026	0.0004	3139	1624	2381	2510	1250	3464	2470	
5	0.0036	0.0004	3305	1544	2424	2268	1250	3648	2568	
7	0.0001	0.0007	272	1205	739	1384	1250	304	694	
8	0.0004	0.0004	1180	1568	1374	2342	1250	1307	1224	
9	0.0001	0.0065	183	1619	901	623	1250	204	903	
Total			*10000*	*10000*	*10000*	*10006*	*10000*	*10000*	*10000*	**SRS**
SE (30)			**0.041**	**0.054**	**0.045**	**0.064**	**0.057**	**0.048**	**0.044**	**0.090**
SE (10)			**0.055**	**0.031**	**0.034**	**0.043**	**0.032**	**0.052**	**0.034**	**0.134**

Appendix 20. Population R15.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Gain	Loss	Forest Gain	Forest Loss	Average					
1	0.0010	0.0114	5028	6571	5800	6807	2500	6209	6296	
2	0.0041	0.0102	4428	2765	3596	3027	2500	3593	3002	
3	0.0070	0.5071	273	648	460	143	2500	170	602	
4	0.3731	0.0060	270	17	144	24	2500	28	99	
Total			*10000*	*10000*	*10000*	*10001*	*10000*	*10000*	*10000*	**SRS**
SE (Gain)			**0.044**	**0.054**	**0.045**	**0.051**	**0.059**	**0.049**	**0.047**	**0.054**
SE (Loss)			**0.120**	**0.110**	**0.113**	**0.120**	**0.158**	**0.118**	**0.111**	**0.133**

Appendix 21. Population R16.

Stratum	P_h		Optimal			Proportional	Equal	Bethel	SSW	
	Reforestation	Deforestation	Reforestation	Deforestation	Average					
1	0.0267	0.4133	1171	3557	2364	211	1250	1417	2484	
2	0.0125	0.0324	118	187	152	31	1250	154	146	
3	0.0084	0.0084	373	371	372	119	1250	470	348	
4	0.0759	0.1607	57	78	67	6	1250	40	64	
5	0.0133	0.0133	41	41	41	11	1250	60	39	
6	0.0132	0.0132	2104	2091	2098	536	1250	2633	1963	
7	0.0001	0.0001	3248	3228	3238	8932	1250	4070	3030	
8	0.3930	0.0058	2887	447	1667	172	1250	1155	1927	
		Total	*9999*	*10000*	*9999*	*10018*	*10000*	*9999*	*10001*	**SRS**
		SE (Reforestation)	**0.029**	**0.046**	**0.031**	**0.075**	**0.041**	**0.033**	**0.031**	**0.091**
		SE (Deforestation)	**0.038**	**0.029**	**0.031**	**0.080**	**0.044**	**0.035**	**0.032**	**0.099**

7.3. Constructed matrices of the experimental design

Appendix 22. Constructed matrix for population 1c.

Population 1c	Reference				**Total**
		a	**b**	**c**	
Map strata	**1**	0.0060	0.0001	0.0039	0.0100
	2	0.0001	0.0300	0.0199	0.0500
	3	0.0039	0.0199	0.9162	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 23. Constructed matrix for population 1d.

Population 1d	Reference				**Total**
		a	**b**	**c**	
Map strata	**1**	0.0060	0.0039	0.0001	0.0100
	2	0.0039	0.0300	0.0161	0.0500
	3	0.0001	0.0161	0.9238	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 24. Constructed matrix for population 2c.

Population 2c	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0001	0.0039	0.0100
	2	0.0001	0.0425	0.0282	0.0708
	3	0.0039	0.0074	0.9079	0.9192
Total		0.0100	0.0500	0.9400	1.0000

Appendix 25. Constructed matrix for population 2d.

Population 2d	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0039	0.0001	0.0100
	2	0.0039	0.0425	0.0244	0.0708
	3	0.0001	0.0036	0.9155	0.9192
Total		0.0100	0.0500	0.9400	1.0000

Appendix 26. Constructed matrix for population 3c.

Population 3c	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0001	0.0039	0.0100
	2	0.0001	0.0300	0.0052	0.0353
	3	0.0039	0.0199	0.9309	0.9547
Total		0.0100	0.0500	0.9400	1.0000

Appendix 27. Constructed matrix for population 3d.

Population 3d	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0039	0.0001	0.0100
	2	0.0039	0.0300	0.0014	0.0353
	3	0.0001	0.0161	0.9385	0.9547
Total		0.0100	0.0500	0.9400	1.0000

Appendix 28. Constructed matrix for population 4c.

Population 4c	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0001	0.0039	0.0100
	2	0.0001	0.0425	0.0074	0.0500
	3	0.0039	0.0074	0.9287	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 29. Constructed matrix for population 4d.

Population 4d	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0039	0.0001	0.0100
	2	0.0039	0.0425	0.0036	0.0500
	3	0.0001	0.0036	0.9363	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 30. Constructed matrix for population 5c.

Population 5c	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0001	0.0056	0.0142
	2	0.0001	0.0300	0.0199	0.0500
	3	0.0014	0.0199	0.9145	0.9358
Total		0.0100	0.0500	0.9400	1.0000

Appendix 31. Constructed matrix for population 5d.

Population 5d	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0056	0.0001	0.0142
	2	0.0014	0.0300	0.0186	0.0500
	3	0.0001	0.0144	0.9213	0.9358
Total		0.0100	0.0500	0.9400	1.0000

Appendix 32. Constructed matrix for population 6c.

Population 6c	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0001	0.0056	0.0142
	2	0.0001	0.0425	0.0282	0.0708
	3	0.0014	0.0074	0.9062	0.9150
Total		0.0100	0.0500	0.9400	1.0000

Appendix 33. Constructed matrix for population 6d.

Population 6d	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0056	0.0001	0.0142
	2	0.0014	0.0425	0.0269	0.0708
	3	0.0001	0.0019	0.9130	0.9150
Total		0.0100	0.0500	0.9400	1.0000

Appendix 34. Constructed matrix for population 7c.

Population 7c	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0001	0.0056	0.0142
	2	0.0001	0.0300	0.0052	0.0353
	3	0.0014	0.0199	0.9292	0.9505
Total		0.0100	0.0500	0.9400	1.0000

Appendix 35. Constructed matrix for population 7d.

Population 7d	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0056	0.0001	0.0142
	2	0.0014	0.0300	0.0039	0.0353
	3	0.0001	0.0144	0.9360	0.9505
Total		0.0100	0.0500	0.9400	1.0000

Appendix 36. Constructed matrix for population 8c.

Population 8c	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0001	0.0056	0.0142
	2	0.0001	0.0425	0.0074	0.0500
	3	0.0014	0.0074	0.9270	0.9358
Total		0.0100	0.0500	0.9400	1.0000

Appendix 37. Constructed matrix for population 8d.

Population 8d	Reference				Total
Map strata		a	b	c	
	1	0.0085	0.0056	0.0001	0.0142
	2	0.0014	0.0425	0.0061	0.0500
	3	0.0001	0.0019	0.9338	0.9358
Total		0.0100	0.0500	0.9400	1.0000

Appendix 38. Constructed matrix for population 9c.

Population 9c	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0001	0.0010	0.0071
	2	0.0001	0.0300	0.0199	0.0500
	3	0.0039	0.0199	0.9191	0.9429
Total		0.0100	0.0500	0.9400	1.0000

Appendix 39. Constructed matrix for population 9d.

Population 9d	Reference				Total
Map strata		a	b	c	
	1	0.0060	0.0010	0.0001	0.0071
	2	0.0039	0.0300	0.0161	0.0500
	3	0.0001	0.0190	0.9238	0.9429
Total		0.0100	0.0500	0.9400	1.0000

Appendix 40. Constructed matrix for population 10c.

Population 10c	Reference			Total
		a	b	c	
Map strata	1	0.0060	0.0001	0.0010	0.0071
	2	0.0001	0.0425	0.0282	0.0708
	3	0.0039	0.0074	0.9108	0.9221
Total		0.0100	0.0500	0.9400	1.0000

Appendix 41. Constructed matrix for population 10d.

Population 10d	Reference				Total
		a	b	c	
Map strata	1	0.0060	0.0010	0.0001	0.0071
	2	0.0039	0.0425	0.0244	0.0708
	3	0.0001	0.0065	0.9155	0.9221
Total		0.0100	0.0500	0.9400	1.0000

Appendix 42. Constructed matrix for population 11c.

Population 11c	Reference				Total
		a	b	c	
Map strata	a	0.0060	0.0001	0.0010	0.0071
	b	0.0001	0.0300	0.0052	0.0353
	c	0.0039	0.0199	0.9338	0.9576
Total		0.0100	0.0500	0.9400	1.0000

Appendix 43. Constructed matrix for population 11d.

Population 11d	Reference				Total
		a	b	c	
Map strata	1	0.0060	0.0010	0.0001	0.0071
	2	0.0039	0.0300	0.0014	0.0353
	3	0.0001	0.0190	0.9385	0.9576
Total		0.0100	0.0500	0.9400	1.0000

Appendix 44. Constructed matrix for population 12c.

Population 12c		Reference			Total
		a	b	c	
Map strata	1	0.0060	0.0001	0.0010	0.0071
	2	0.0001	0.0425	0.0074	0.0500
	3	0.0039	0.0074	0.9316	0.9429
Total		0.0100	0.0500	0.9400	1.0000

Appendix 45. Constructed matrix for population 12d.

Population 12d		Reference			Total
		a	b	c	
Map strata	1	0.0060	0.0010	0.0001	0.0071
	2	0.0039	0.0425	0.0036	0.0500
	3	0.0001	0.0065	0.9363	0.9429
Total		0.0100	0.0500	0.9400	1.0000

Appendix 46. Constructed matrix for population 13c.

Population 13c		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0001	0.0014	0.0100
	2	0.0001	0.0300	0.0199	0.0500
	3	0.0014	0.0199	0.9187	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 47. Constructed matrix for population 13d.

Population 13d		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0014	0.0001	0.0100
	2	0.0014	0.0300	0.0186	0.0500
	3	0.0001	0.0186	0.9213	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 48. Constructed matrix for population 14c.

Population 14c		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0001	0.0014	0.0100
	2	0.0001	0.0425	0.0282	0.0708
	3	0.0014	0.0074	0.9104	0.9192
Total		0.0100	0.0500	0.9400	1.0000

Appendix 49. Constructed matrix for population 14d.

Population 14d		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0014	0.0001	0.0100
	2	0.0014	0.0425	0.0269	0.0708
	3	0.0001	0.0061	0.9130	0.9192
Total		0.0100	0.0500	0.9400	1.0000

Appendix 50. Constructed matrix for population 15c.

Population 15c		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0001	0.0014	0.0100
	2	0.0001	0.0300	0.0052	0.0353
	3	0.0014	0.0199	0.9334	0.9547
Total		0.0100	0.0500	0.9400	1.0000

Appendix 51. Constructed matrix for population 15d.

Population 15d		Reference			Total
		a	b	c	
Map strata	1	0.0085	0.0014	0.0001	0.0100
	2	0.0014	0.0300	0.0039	0.0353
	3	0.0001	0.0186	0.9360	0.9547
Total		0.0100	0.0500	0.9400	1.0000

Appendix 52. Constructed matrix for population 16c.

Population 16c	Reference				Total
		a	b	c	
Map strata	1	0.0085	0.0001	0.0014	0.0100
	2	0.0001	0.0425	0.0074	0.0500
	3	0.0014	0.0074	0.9312	0.9400
Total		0.0100	0.0500	0.9400	1.0000

Appendix 53. Constructed matrix for population 16d.

Population 16d	Reference				Total
		a	b	c	
Map strata	1	0.0085	0.0014	0.0001	0.0100
	2	0.0014	0.0425	0.0061	0.0500
	3	0.0001	0.0061	0.9338	0.9400
Total		0.0100	0.0500	0.9400	1.0000

7.4. R code for Bethel method allocation

The following code provides an example of how to use the Bethel function in R. The specific example refers to the R13 case, which includes the data from Table 7 Chen et al. (2021). This dataset contains 7 strata, and the analysis focuses on two target classes: class a, representing Deforestation, and class b, representing Degradation no Regeneration.

```
# Install devtools package to enable GitHub package installations

install.packages("devtools")

# Install SamplingStrata package from GitHub

devtools::install_github("barcaroli/SamplingStrata")

# Load the SamplingStrata library
```

```
library(SamplingStrata)

# Read the dataset from the specified CSV file

dataset <- read.csv("C:/Users/Fernanda/Documents/SUNY - ESF/TESIS/5. QUINTO EJERCICIO/Chen_etal_2021_Table_7.1_R.csv")

# Select relevant columns from the dataset

data <- subset(dataset, select = c(stratum, N, M1, M2, S1, S2, cens, cost, DOM1))

# Rename columns for clarity

colnames(data) <- c("stratum", "N", "M1", "M2", "S1", "S2", "cens", "cost", "DOM1")

# Define the Coefficient of Variation (CV) requirements for the domain

cv <- data.frame(DOM = "DOM1", CV1 = 0.05, CV2 = 0.05, domainvalue = 1)

errors <- cv[1, 1:4]

# Determine the optimal sample size allocation across strata

n <- bethel(data, errors, printa = TRUE, maxiter = 1000)

# Print the resulting sample sizes for each stratum

n
```

7.5. CVS file example

The following example provides an example of how to create the CVS file needed for Bethel analysis in R. The specific example refers to the R13 case, which includes the data from

Table 7 Chen et al. (2021). This dataset contains 7 strata, and the analysis focuses on two target classes: class a, representing Deforestation, and class b, representing Degradation no Regeneration.

```
stratum,N,M1,M2,S1,S2,cens,cost,DOM1

1,33,0.6060,0.0303,0.4886,0.1714,0,1,1

2,91,0.0109,0.1098,0.1042,0.3127,0,1,1

3,461,0.0021,0.3253,0.0465,0.4685,0,1,1

4,132,0.0075,0.6818,0.0867,0.4657,0,1,1

5,291,0.0343,0.0687,0.1821,0.2529,0,1,1

6,3491,0.0002,0.0085,0.0169,0.0923,0,1,1

7,5493,0.0001,0.0001,0.0134,0.0134,0,1,1
```

8. VITA OR RESUME

MARIA FERNANDA GONZALEZ MONTES

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QUALIFICATION SUMMARY

Dynamic forest resources professional with a Master's in Forest Resources Management and a Bachelor's in Forest Engineering. Demonstrated expertise in research and teaching at SUNY – ESF, alongside hands-on experience in social reforestation, wildlife monitoring, and ecosystem restoration projects. Recognized for leadership in campus organizations, recipient of notable grants and awards, and proficient in software tools. Fluent in Spanish and English, with basic German skills.

EDUCATION

Master of Science, Forest Resources Management *Aug 2024*

State University of New York College of Environmental Science and Forestry (ESF), Syracuse, NY | GPA: 3.84/4.00

Bachelor of Science, Forest Engineering *June 2022*

Universidad Autónoma Chapingo (UACh), Texcoco de Mora, Estado de México, MX | GPA: 3.70/4.00

RESEARCH EXPERIENCE

Research Project Assistant *August 2022 – August 2024*

Department of Sustainable Resources Management, SUNY – ESF, Syracuse, NY

- Evaluation and comparison of various multi-decision optimal allocation methods in stratified sampling to enhance precision in forest area estimation (e.g., deforestation and degradation). Designing recommendations and effective practices for the implementation of allocation methods in forest monitoring. Proposal of strategies and lessons learned based on the analysis, to contribute to sustainable forest resource management.
- Development of training materials for estimating uncertainties of activity data.

TEACHING EXPERIENCE

Teaching Assistant *January 2023 – May 2023*

Department of Sustainable Resources Management, SUNY – ESF, Syracuse, NY

- *Intro/Probability&Stats* (APM 391; Spring 2023): Organized and led weekly lab sessions in Minitab, managed grading, occasional lecturer, and advised students on independent research projects or final projects.

PROFESSIONAL EXPERIENCE

Intern *09/2024 – Present*

International Union of Forest Research Organizations (IUFRO), Vienna, Austria

Technical Advisor in Social Reforestation Project *06/2022 - 12/2022*

Pentathlón Deportivo Militarizado Universitario, Oaxaca, México

- Advice on the urban reforestation process, establishment, and maintenance of a nursery, staff training, and environmental awareness.

Internship in Wildlife Monitoring *02/2022 - 05/2022*

"Inotawa Expeditions" Puerto Maldonado, Perú

- Conducting inventories and catalogs of local wildlife. Implementation and management of camera traps to monitor mammal presence and behavior in their natural environment. Direct monitoring of species through field observation. Data collection on behavior and preferred habitats. Detailed analysis to calculate relative abundance indices. Designing effective conservation strategies.

Technical Advisor in Ecosystem Restoration Project *02/2021 - 08/2021*
Cooperative Society "Lagarto Real" Oaxaca, México

- Technical advice on Red Mangrove (*Rhizophora mangle*) in the nursery stage and the establishment of new technologies in the reforestation process. Management and creation of marketing campaigns to facilitate the dissemination of content. Preparation of progress reports for the Comisión Nacional Forestal (CONAFOR).

GRANTS AND AWARDS

Cline Silviculture Scholarship *08/2023 - 05/2024*
SUNY – ESF, Syracuse, NY

Graduate Assistantship: Quantifying Uncertainty Estimates and Risk for Carbon Accounting (QUERCA) at SUNY – ESF, Syracuse, NY *08/2022 - 08/2024*
SilvaCarbon in collaboration with USDA Forest Service - International Programs, USAID, The World Bank and FAO

Academic Excellence Program *08/2017 - 06/2022*
UACh, Texcoco, Estado de México, MX

LEADERSHIP EXPERIENCE & CAMPUS SERVICE

International Forestry Students' Association (IFSA)
Head of Carbon and Sustainability Sub-Commission *10/2023 – Present*

- Monitoring IFSA's emissions during events and travels. Tracking Officials' travels. Implementing actions aligned with IFSA's 2030 Sustainability Strategy, organized into 5 goals. Prepare carbon emissions reports in terms of transportation.

Member *01/2022 – Present*

- Member of the Organizing Committee of the Latin American Regional Meeting 2024.

Head of Summer School at IFSA LC CHAUPEA *12/2022 – 09/2023*

- Organize and lead courses enhancing students' practical skills and technical competencies. Deliver educational talks and facilitate discussions with students from various universities on topics of interest, promoting the exchange of global research findings and improving their English skills.

Forestry Club, Society of American Foresters (SAF) Chapter *08/2023 - 05/2024*
SUNY – ESF, Syracuse, NY
Secretary

- Maintain meeting minutes and a permanent record of all meetings. Handle all correspondence for the club and manage documentation of materials sent and received. Publish meeting minutes for all members. Compile and distribute an email list of members. Manage attendance at club events.

Fire Management Brigade, UACh, Texcoco de Mora, México *08/2018 - 06/2022*
Member

- Coordination of reforestation and firefighting projects. Social media management.

SKILLS

Computer: ArcGIS, SILVAH, RStudio, IDRISI-TerrSet, QM for Windows V5, Minitab, MS-Office
Languages: Spanish (Native), English (Advanced – C1), German (Basic – A2)