iLTER 2016: Measurement Uncertainty Workshop

Presenter: Janae Csavina

Contributors: Joshua Roberti and Hank Loescher

Will this be the right workshop for you?

This will be an introduction to measurement uncertainty. If you have never done an uncertainty analysis and maybe have never even thought about its relevance to your research but would like to understand its importance and basic examples of its application, this workshop will provide these core topics. If you have meddled in uncertainty analysis and want to verify your understanding, this could also be relevant. However, if your research is focused on modeling or you want to better your experimental design to reduce measurement uncertainty, the other uncertainty workshops will be more relevant. The exercises will focus on sensor based measurements, but applications can include observation based measurements.

What materials you will need?

To get the most out of this workshop, bring a laptop with Microsoft Excel and download the exercise workbook from the website. I will be reviewing the solutions and methods, so while a laptop is not necessary, participating in the exercises will aid in learning. Most applied statistics will be covered and included below for reference, but a basic understanding of Excel formula applications will be beneficial.

What can you expect to come away with from this workshop?

This workshop aims to provide the basis for how to do a basic uncertainty analysis on a measurement system and why a traceable measurement is a necessary element for uncertainty analysis as well as scientific research. Topics will include national and international standards, uncertainty analysis components, combined and expanded uncertainty, translating calibration certificates, field validations, outlier removal, and how to apply all this to different types of measurements.

Here are some helpful things to know:

Background: Uncertainty analysis is internationally standardized through a document called "Evaluation of measurement data – Guide to the expression of uncertainty in measurement" produced by Joint Committee for Guides and Metrology (JCGM) made up of seven international organizations (BIPM, CEI, IFCC, ILAC, ISO, UICPA, UIPPA et OIML). This metrology guide is extensive and but difficult for non-specialists to understand, hence the purpose of this workshop. To do a proper uncertainty analysis, one must start with a traceable measurement.

Application: There are two methods for quantifying uncertainty described in the GUM: Type A uses statistical methods while Type B uses someone else's or previous analysis i.e. published material. An example of Type A is the standard deviation of a measurement while an example of Type B would be a calibration certificate for a sensor or data acquisition system (DAS). This is where traceable measurements come into play; a simple application of traceable measurements is having a sensor calibration certificate that is traceable to a nationally or internationally recognized standard such as the International System of Units (SI) or World Radiometric Center (WRC) and within the valid date range of the calibration certificate. More examples of a standard for non-sensor based measurements include a nationally recognized taxonomic key or internationally accepted protocol with quality metrics quantified for the given key or protocol.

There are two basic components to all measurement uncertainty, a traceable method or sensor calibration and the repeatability of the measurement. There can be other components that contribute to uncertainty such as a data acquisition system (DAS) that takes analogue signals from the sensor or sensor drift over the measurement period. Figure 1 provides an illustration of the components that make up measurement uncertainty.

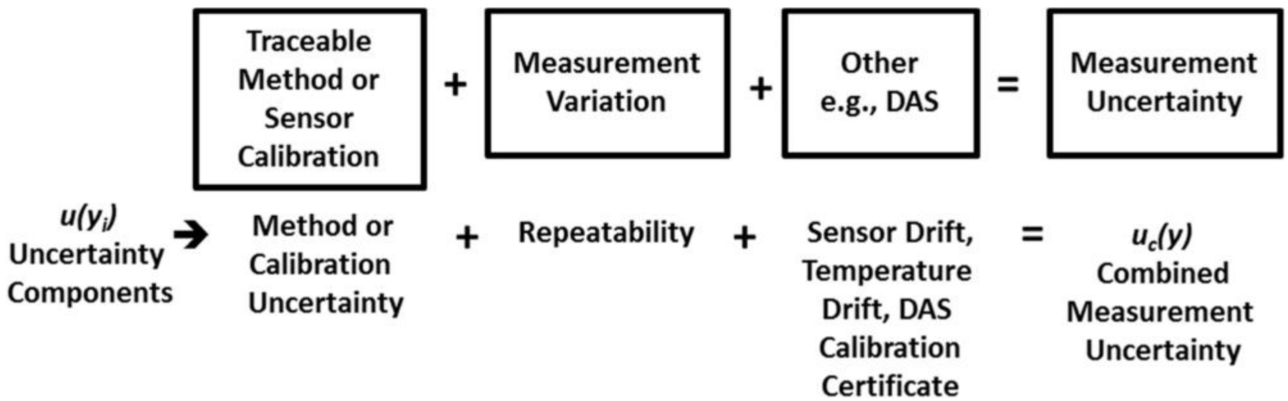

Figure 1. Uncertainty assessment components for a measurement system. The boxes provide basic concepts and below the boxes are the typical sources of the uncertainty component.

As mentioned above, sensor calibration uncertainty estimates will come from the certificate of the given instrument. Sometimes the certificate quotes an accuracy or tolerance for the sensor, in which case, equations 1 (a) and 1 (b) can be used to translate these quantities to uncertainty, respectively. Note that when you use someone else's analysis, you have to assume a degrees of freedom for the quoted uncertainty. Because many times the coverage factor is assumed to be 2, it is acceptable to use a degrees of freedom of 100 to represent the confidence in that estimate. Repeatability can be the standard deviation (Equation 2) of measurements taken under repeatable conditions if one measurement represents the result to be reported. However, repeatability can also be the standard error (Equation 3) of a measurement if an average is taken to represent the measurement. For example, if a measurement is taken once a second and the results are reported as minute averages (Equation 4) of the 60 measurements, the standard error of the 60 measurements represents the repeatability. Equation 5 shows how these components will be added together in quadrature. This equation is simplified from the law of propagation of uncertainty based on the first-order Taylor series approximation. The assumptions for simplification include a normal distribution in the measurements, the uncertainty components are uncorrelated, and all uncertainty components are in the final measurement units. The level of confidence for the combined measurement uncertainty, $u_c(y)$, is 68%.

To increase the level of confidence in the uncertainty estimate, the combined uncertainty is multiplied by a coverage factor, k_p (Equation 6). The expanded uncertainty, U_p, is often desired to be quoted at a confidence level of 95%. Therefore, p=95 and assuming a normal distribution, $k_{95} \approx 2$. During the workshop, I will review the Welch-Satterthwaite formula (Equation 7) which is an alternative to assuming a coverage factor and properly account for the degrees of freedom in your analysis. The coverage factor then becomes Equation 8.

Other applied topics that will be coved in the exercises include outlier removal using interquartile range (IQR) assumptions (Equations 9 a-e) and validation repeatability (Equation 10).

Equations:

$$u(y_i) = \frac{a}{3} \tag{1a}$$

Where $u(y_i)$ is the uncertainty component and a is the accuracy provided in a calibration certificate.

$$u(y_i) = \frac{t}{\sqrt{3}} \tag{1b}$$

Where $u(y_i)$ is the uncertainty component and a is the accuracy provided in a calibration certificate.

$$u(y_i) = s(y_i) = \left[\frac{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2}{(n-1)}\right]^{\frac{1}{2}} \tag{2}$$

Where $u(y_i)$ is the uncertainty component, $s(y_i)$ is the standard deviation of y_i, Y_i is the input quantity, $\bar{Y}_i$ is the average of the input quantities, and n is the number of independent observations of Y_i.

$$u(y_i) = s(\bar{y}_i) = \frac{s(y_i)}{n^{1/2}} \tag{3}$$

Where $u(y_i)$ is the uncertainty component, $s(\bar{y}_i)$ is the standard deviation of the mean ($\bar{Y}_i$) or standard error, $s(y_i)$ is the standard deviation of y_i, and n is the number of independent observations of Y_i.

$$y_i = \bar{Y}_i = \frac{1}{n}\sum_{i=1}^{n} Y_i \tag{4}$$

Where Y_i is the input quantity, n is the number of independent observations of Y_i, and y_i is the reported resultant or input estimate.

$$u_c(y) = \left[\sum_{i=1}^{n} u_i^2(y)\right]^{\frac{1}{2}} \quad (5)$$

Where $u_c(y)$ is the combined uncertainty, $u_i(y)$ are the independent uncertainty components that contribute to measurement uncertainty.

$$U_p = k_p\, u_c(y) \quad (6)$$

Where the p subscript denotes the confidence level, U_p is expanded uncertainty, k_p is the coverage factor, and $u_c(y)$ is the combined uncertainty.

$$\nu_e \quad = \frac{u_c^4(y)}{\sum_{i=1}^{n} \frac{u_i^4(y)}{\nu_i}} \quad (7)$$

Where ν_{eff} is the effective degrees of freedom, $u_c(y)$ is the combined uncertainty, $u_i(y)$ are the independent uncertainty components, and ν_i is the degrees of freedom for the uncertainty component (typically ν=n-1).

$k_p = t_p(\nu_{eff})$ (8)

Where the p subscript denotes the confidence level, k_p is the coverage factor, and $t_p(\nu_{eff})$ is based on the effective degrees of freedom in Equation 7 and obtained from Table G.2 in the *GUM* or other t-distribution sources.

$Q1(y_i)$ = median(y_i) – 0.6745 s(y_i) (9a)
$Q3(y_i)$ = median(y_i) – 0.6745 s(y_i) (9b)
IQR(y_i) = Q3(y_i) – Q1(y_i) (9c)
High Outlier(y_i) = Q3(y_i) + 1.5*IQR(y_i) (9d)
Low Outlier(y_i) = Q1(y_i) - 1.5*IQR(y_i) (9e)

Where Q1(y_i) is first quartile, Q3(y_i) is the third quartile, median(y_i) is a quantity lying at the midpoint of a frequency distribution, s(y_i) is the standard deviation, IQR(y_i) is the interquartile range, and high outlier (y_i) and low outlier(y_i) denotes non-outlier range for observed quantities of y_i.

$$u(y_i) = \left[\frac{\sum_{i=1}^{n}(Y_i - V_i)^2}{(\mathrm{n}-1)}\right]^{\frac{1}{2}} \quad (10)$$

Where $u(y_i)$ is the uncertainty component for validation repeatability, Y_i is the input quantity for the out-of-calibration sensor, V_i is the input quantity for the in-calibration sensor used in the validation, and n is the number of independent observations.

References:

ISO (International Organization for Standardization), 1995: Guide to the Expression of Uncertainty in Measurement. International Organization for Standardization.

Taylor, B. N., and C. E. Kuyatt, (1994) Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Technical Note 1297.